In[∘]:= ClearAll["Global`*"]

- Preliminaries
- We allow for arbitrary μ and α
- Throughout, we operate with interior parameter values $\mu, \alpha \in (0,1)$. Including the boundary cases is straightforward.
- We set $\delta = 1$ for ease of exposition.

```
In[∘]:= (* Initialize Utilities *)

In[∘]:= ulow = u;

In[∘]:= uhigh = ulow + Δ;

In[∘]:= (* Expected Utility *)

In[∘]:= exput = ulow + μ (uhigh - ulow)

Out[∘]= u + Δ μ

In[∘]:= (* Predicted Expected Utility of Motivated biased Consumers *)

In[∘]:= expmot = FullSimplify[μ uhigh + (1 - μ) (α uhigh + (1 - α) ulow) ]

Out[∘]= u + Δ (α + μ - α μ)

In[∘]:= (* Predicted Expected Utility of Unmotivated biased Consumers *)

In[∘]:= expunmot = FullSimplify[μ (α ulow + (1 - α) uhigh) + (1 - μ) ulow]

Out[∘]= u - (-1 + α) Δ μ

In[∘]:= assumptions = 1 > α > 0 && 1 > μ > 0 && u > 0 && Δ > 0 && c ∈ ℝ;
```

- Baseline Model -- Preparation for Welfare & Demand

```
In[∘]:= (* Initiate Demand by motivated consumers, motd, and unmotivated consumers unmotd *)

In[∘]:= (* Assume that consumers located at point x suffer "transport costs" equal to x;
        on any line of length L > 0 there is a mass L of consumers *)

In[∘]:= (* Get cutoff consumers of each group - motivated and unmotivated *)

In[∘]:= xmot = x /. Solve[expmot - p - x == 0, x][[1]];
        xunmot = x /. Solve[expunmot - p - x == 0, x][[1]];

In[∘]:= (* Demand follows from cutoff consumer multiplied by proportion of relevant group *)

In[∘]:= (* motivated consumers: μ *)

In[∘]:= motd = FullSimplify[μ * xmot]

Out[∘]= μ (-p + u + Δ (α + μ - α μ))

In[∘]:= (* unmotivated consumers: (1-μ) *)

In[∘]:= unmotd = FullSimplify[(1 - μ) * xunmot]

Out[∘]= (-1 + μ) (p - u + (-1 + α) Δ μ)

In[∘]:= (* For Welfare we require the total transport costs paid by
         consumers of each type which again depend on the cutoff consumer *)
```

In[ ]:=
```
(* Transport Costs are equal to mass of consumer type
 time times integral of transport costs up to cutoff consumer *)
```

In[ ]:=
```
mottc = μ Integrate[x, {x, 0, xmot}]
```

Out[ ]= $\frac{1}{2}\mu\,(-p+u+\alpha\Delta+\Delta\mu-\alpha\Delta\mu)^2$

In[ ]:=
```
unmottc = (1 - μ) Integrate[x, {x, 0, xunmot}]
```

Out[ ]= $\frac{1}{2}(1-\mu)\,(-p+u+\Delta\mu-\alpha\Delta\mu)^2$

In[ ]:=
```
(* Consumer Welfare of
 motivated: Demand times (actual exput minus price) net of total transport costs *)
```

In[ ]:=
```
motcw = FullSimplify[motd * (exput - p) - mottc, assumptions]
```

Out[ ]= $-\frac{1}{2}\mu\,\left(p-u-\alpha\Delta+(-1+\alpha)\,\Delta\mu\right)\,\left(-p+u+\Delta\,\left(\alpha\,(-1+\mu)+\mu\right)\right)$

In[ ]:=
```
(* Same for unmotivated consumers *)
```

In[ ]:=
```
unmotcw = FullSimplify[unmotd * (exput - p) - unmottc, assumptions]
```

Out[ ]= $\frac{1}{2}(-1+\mu)\,\left(p-u+(-1+\alpha)\,\Delta\mu\right)\,\left(-p+u+(1+\alpha)\,\Delta\mu\right)$

- Strategy 1: Full Targeting

In[ ]:=
```
(* Full targeting caters to some
  unmotivated consumers and hence requires p < expunmot *)
```

In[ ]:=
```
(* Demand is then equal to the demand of both groups *)
```

In[ ]:=
```
dfull = FullSimplify[motd + unmotd]
```

Out[ ]= $-p+u+\Delta\mu$

In[ ]:=
```
(* Solve Maximization Problem conditional on targeting strategy *)
```

In[ ]:=
```
tempsol = FullSimplify[Maximize[(p - c) * dfull, p]]
```

Out[ ]= $\left\{\frac{1}{4}(-c+u+\Delta\mu)^2,\ \left\{p\to\frac{1}{2}(c+u+\Delta\mu)\right\}\right\}$

In[ ]:=
```
(* Use solution to characterize profits, price, quantity, CW *)
```

In[ ]:=
```
proffull = tempsol[[1]];
pfull = p /. tempsol[[2]];
qfull = FullSimplify[dfull /. p → pfull];
cwfull = FullSimplify[motcw + unmotcw /. p → pfull, {1 > μ > 0, Δ > 0, 1 > α > 0}];
```

In[ ]:=
```
(* Print Solutions *)
```

In[ ]:=
```
FullSimplify[{qfull, cwfull}, assumptions]
```

Out[ ]= $\left\{\frac{1}{2}(-c+u+\Delta\mu),\ \frac{1}{8}\left(c^2+u^2+2u\Delta\mu+\Delta^2\mu\,\left(4\alpha^2\,(-1+\mu)+\mu\right)-2c\,(u+\Delta\mu)\right)\right\}$

In[ ]:=
```
(* Obtain cost threshold such that solution is interior,
i.e. such that price equals maximal price such that both consumer groups purchase *)
```

In[∘]:= Solve[pfull == expunmot, c]

Out[∘]= $\{\{c \to u + \Delta\,\mu - 2\,\alpha\,\Delta\,\mu\}\}$

```
In[∘]:= (* Unique cutoff cost *)

In[∘]:= cfullhigh = FullSimplify[c /. Solve[pfull == expunmot, c][[1]]];
```

- Strategy 2: Only Motivated

```
In[∘]:= (* This strategy requires p > expunmot,
but p < expmot so that some motivated consumers purchase *)

In[∘]:= (* Demand in this case is composed only of the group of motivated consumers *)

In[∘]:= dexcl = motd
```

Out[∘]= $\mu\,(-p + u + \Delta\,(\alpha + \mu - \alpha\,\mu))$

```
In[∘]:= (* Solve Maximization Problem conditional on Targeting Strategy *)

In[∘]:= tempsol = FullSimplify[Maximize[(p - c) * dexcl, p], assumptions]
```

Out[∘]= $\left\{\frac{1}{4}\,\mu\,(-c + u + \Delta\,(\alpha + \mu - \alpha\,\mu))^2,\ \left\{p \to \frac{1}{2}\,(c + u + \Delta\,(\alpha + \mu - \alpha\,\mu))\right\}\right\}$

```
In[∘]:= (* Use solution to characterize profits, price, quantity, CW *)

In[∘]:= profexcl = tempsol[[1]];
pexcl = p /. tempsol[[2]];
qexcl = FullSimplify[dexcl /. p → pexcl];
cwexcl = FullSimplify[motcw /. p → pexcl];

In[∘]:= (* Print solutions *)

In[∘]:= FullSimplify[{qexcl, cwexcl}, assumptions]
```

Out[∘]= $\left\{\frac{1}{2}\,\mu\,(-c + u + \Delta\,(\alpha + \mu - \alpha\,\mu)),\ -\frac{1}{8}\,\mu\,\left(c - u - \alpha\,\Delta + (-1 + \alpha)\,\Delta\,\mu\right)\,\left(-c + u + \Delta\,\left(3\,\alpha\,(-1 + \mu) + \mu\right)\right)\right\}$

```
In[∘]:= (* Get cost threshold such that solution is interior,
i.e. price such that only motivated biased consumers purchase *)

In[∘]:= Solve[pexcl == expunmot, c]
```

Out[∘]= $\{\{c \to u - \alpha\,\Delta + \Delta\,\mu - \alpha\,\Delta\,\mu\}\}$

```
In[∘]:= cexcllow = c /. FullSimplify[Solve[pexcl == expunmot, c]][[1]];
```

- Characterization of Optimal Pricing -- derive cost threshold
- Comparison of Full Targeting and Exclusive Targeting

```
In[∘]:= FullSimplify[Solve[profexcl == proffull, c], assumptions]
```

Out[∘]= $\left\{\left\{c \to u + \Delta\,\left(\mu - \alpha\,\left(\sqrt{\mu} + \mu\right)\right)\right\},\ \left\{c \to u + \Delta\,\left(\alpha\,\left(\sqrt{\mu} - \mu\right) + \mu\right)\right\}\right\}$

```
In[∘]:= c1temp = c /. FullSimplify[Solve[profexcl == proffull, c], assumptions][[1]];
c2temp = c /. FullSimplify[Solve[profexcl == proffull, c], assumptions][[2]];

In[∘]:= FullSimplify[c1temp < exput < c2temp, assumptions]
```

Out[∘]= True

```
In[∘]:= (* The relevant cutoff must be the lower root, i.e. the first solution *)
```

In[∘]:=
```
ctilde = c1temp;
```

- Verify that cutoff leads to interior solutions

In[∘]:=
```
FullSimplify[cfullhigh > ctilde, assumptions]
```
Out[∘]=
```
True
```

In[∘]:=
```
FullSimplify[ctilde > cexcllow, assumptions]
```
Out[∘]=
```
True
```

In[∘]:=
```
(* Interior solution ensured, pricing determined *)
```

- Cooling Off
- ===========
- Repeat same exercise as previously, only adjusting for different weights of the groups

In[∘]:=
```
(* Get cutoff consumers of each group - motivated and unmotivated *)
```

In[∘]:=
```
xmotco = x /. Solve[expmot - p - x == 0, x][[1]];
xunmotco = x /. Solve[expunmot - p - x == 0, x][[1]];
```

In[∘]:=
```
(* Use cutoff consumers to determine
 demand from each group by accounting for weight *)
(* Demand by motivated consumers is that of twice motivated consumers *)
(* Demand by unmotivated consumers here
 comprises those unmotivated in at least one period *)
```

In[∘]:=
```
motdco = μ^2 xmotco;
unmotdco = ((1 - μ) + μ (1 - μ)) xunmotco;
```

In[∘]:=
```
(* For Welfare we require the total transport costs
 paid by each type which again depend on the cutoff consumer *)
```

In[∘]:=
```
mottcco = μ^2 Integrate[x, {x, 0, xmotco}];
unmottcco = (μ (1 - μ) + (1 - μ)) Integrate[x, {x, 0, xunmotco}];
```

In[∘]:=
```
(* CW evaluated using actual expected utility exput *)
```

In[∘]:=
```
motcwco = FullSimplify[motdco (exput - p) - mottcco];
unmotcwco = FullSimplify[unmotdco * (exput - p) - unmottcco];
```

- Strategy 1 : Exclusive Targeting, p > expunmot

In[∘]:=
```
dexclco = FullSimplify[motdco];
```

In[∘]:=
```
(* Print for Demand Function *)
```

In[∘]:=
```
FullSimplify[dexclco]
```
Out[∘]= $\mu^2\ (-p + u + \Delta\ (\alpha + \mu - \alpha\,\mu))$

In[∘]:=
```
(* Solve Maximization Problem conditional on Targeting Strategy *)
```

In[∘]:=
```
tempsol = FullSimplify[Maximize[(p - c) * dexclco, p], assumptions]
```
Out[∘]= $\left\{\frac{1}{4}\mu^2\ (-c + u + \Delta\ (\alpha + \mu - \alpha\,\mu))^2,\ \left\{p \to \frac{1}{2}\ (c + u + \Delta\ (\alpha + \mu - \alpha\,\mu))\right\}\right\}$

In[∘]:=
```
(* Use solution to characterize profits, price, quantity, CW *)
```

In[∘]:=
```
profexclco = tempsol[[1]];
pexclco  = p /. tempsol[[2]];
qexclco = FullSimplify[dexclco /. p → pexclco];
cwexclco = FullSimplify[motcwco /. p → pexclco];
```

In[∘]:=
```
(* Print Solutions *)
```

In[∘]:=
```
FullSimplify[{qexclco, cwexclco}, assumptions]
```

Out[∘]= $\left\{\frac{1}{2}\mu^2\,(-c+u+\Delta\,(\alpha+\mu-\alpha\,\mu)),\ -\frac{1}{8}\mu^2\,\left(c-u-\alpha\,\Delta+\left(-1+\alpha\right)\Delta\,\mu\right)\,\left(-c+u+\Delta\,\left(3\,\alpha\,\left(-1+\mu\right)+\mu\right)\right)\right\}$

In[∘]:=
```
(* Get cost thresholds so that solution is interior *)
```

In[∘]:=
```
Solve[pexclco == expunmot, c]
```

Out[∘]= $\{\{c\to u-\alpha\,\Delta+\Delta\,\mu-\alpha\,\Delta\,\mu\}\}$

In[∘]:=
```
cexcllowco = c /. FullSimplify[Solve[pexclco == expunmot, c]][[1]];
```

- Strategy 2: Full Targeting

In[∘]:=
```
dfullco = FullSimplify[motdco + unmotdco];
```

In[∘]:=
```
(* Print for Demand Function *)
```

In[∘]:=
```
FullSimplify[dfullco]
```

Out[∘]= $-p+u+\Delta\,\left(1+\alpha\,\left(-1+\mu\right)\right)\,\mu$

In[∘]:=
```
(* Solve Maximization Problem conditional on Targeting Strategy *)
```

In[∘]:=
```
tempsol = FullSimplify[Maximize[(p - c) * dfullco, p], assumptions]
```

Out[∘]= $\left\{\frac{1}{4}\,\left(c-u+\Delta\,\mu\,\left(-1+\alpha-\alpha\,\mu\right)\right)^2,\ \left\{p\to\frac{1}{2}\,\left(c+u+\Delta\,\left(1+\alpha\,\left(-1+\mu\right)\right)\,\mu\right)\right\}\right\}$

In[∘]:=
```
(* Use solution to characterize profits, price, quantity, CW *)
```

In[∘]:=
```
proffullco = tempsol[[1]];
pfullco = p /. tempsol[[2]];
qfullco = FullSimplify[dfullco /. p → pfullco];
cwfullco = FullSimplify[motcwco + unmotcwco /. p → pfullco, assumptions];
```

In[∘]:=
```
(* Print Solutions *)
```

In[∘]:=
```
FullSimplify[{qfullco, cwfullco}, assumptions]
```

Out[∘]= $\left\{\frac{1}{2}\,\left(-c+u+\Delta\,\left(1+\alpha\,\left(-1+\mu\right)\right)\,\mu\right),\ \frac{1}{8}\,\left(c^2+u^2+2\,u\,\Delta\,\mu\,\left(1+\alpha-\alpha\,\mu\right)-2\,c\,\left(u+\Delta\,\mu\,\left(1+\alpha-\alpha\,\mu\right)\right)+\Delta^2\,\mu^2\,\left(1+\alpha\,\left(-1+\mu\right)\,\left(-2+\alpha\,(7+\mu)\right)\right)\right)\right\}$

In[∘]:=
```
(* Show that cwfullco equates to expression in PDF *)
```

In[∘]:=
```
FullSimplify[cwfullco ==
  1/8 ((u - c) (u + 2 (1 + (1 - μ) α) μ Δ - c) + (1 + (1 - μ) α (2 - (7 + μ) α)) μ^2 Δ^2)]
```

Out[∘]= True

In[∘]:=
```
(* Get cost threshold so that solution is interior *)
```

In[∘]:=
```
Solve[pfullco == expunmot, c]
```

Out[∘]= $\left\{\left\{c\to u+\Delta\,\mu-\alpha\,\Delta\,\mu-\alpha\,\Delta\,\mu^2\right\}\right\}$

In[∘]:=
```
cfullhighco = c /. FullSimplify[Solve[pfullco == expunmot, c]][[1]];
```

- Characterize Optimal Targeting

- Comparison of Full and Exclusive Targeting

In[∘]:=
```
FullSimplify[Solve[profexclco == proffullco, c], assumptions]
```

Out[∘]= $\{\{c \to u + \Delta\mu\},\ \{c \to u + (1 - 2\alpha)\,\Delta\mu\}\}$

In[∘]:=
```
c1temp = c /. FullSimplify[Solve[profexclco == proffullco, c], assumptions][[1]];
c2temp = c /. FullSimplify[Solve[profexclco == proffullco, c], assumptions][[2]];
```

In[∘]:=
```
FullSimplify[c1temp > c2temp, assumptions]
```

Out[∘]= True

In[∘]:=
```
(* The relevant cutoff must be the lower root, i.e. the second solution *)
```

In[∘]:=
```
ctildeco = c2temp;
```

In[∘]:=
```
(* Verify that this leads to pricing being interior *)
```

In[∘]:=
```
FullSimplify[cfullhighco > ctildeco, assumptions]
```

Out[∘]= True

In[∘]:=
```
FullSimplify[ctildeco > cexcllowco, assumptions]
```

Out[∘]= True

In[∘]:=
```
(* Interior targeting ensured, so pricing is determined *)
```

- Return Policy

In[∘]:=
```
(* Demand *)
```

In[∘]:=
```
(* First initialize predicted utility in motivated state for unmotivated consumers *)
```

In[∘]:=
```
uunmotmot = FullSimplify[α ulow + (1 - α) uhigh]
```

Out[∘]= $u + \Delta - \alpha\Delta$

In[∘]:=
```
(* Initiate Demand -
 tendering decisions depend on first period predicted utility for high state,
confirmation decisions potentially on utility in low state *)
```

In[∘]:=
```
(* xexclrp: Cutoff such that motivated consumers do not return,
xlowrp: Cutoff such that unmotivated consumers do not return *)
```

In[∘]:=
```
(* xmedrp: Cutoff such that unmotivated consumers tender in the first period *)
```

In[∘]:=
```
xexclrp = x /. Solve[uhigh - p - x == 0, x][[1]];
xmedrp = x /. Solve[uunmotmot - p - x == 0, x][[1]];
xlowrp = x /. Solve[ulow - x - p == 0, x][[1]];
```

In[∘]:=
```
(* Three strategies catering to different people *)
```

```
In[]:= (* Consider first second-period motivated people who in the
         first period correctly predict the utility in the motivated state *)
       (* These are consumers who are motivated in the first period *)
       (* These consumers initially buy provided that uhigh - p - x ≥
        0 where x is their distance *)
       (* We can characterize the demand of these consumers
        and associated transport costs and CW *)

In[]:= excldrp = FullSimplify[μ^2 xexclrp];
       excltcrp = μ^2 Integrate[x, {x, 0, xexclrp}];
       exclcwrp = FullSimplify[excldrp * (uhigh - p) - excltcrp];

In[]:= (* A second group of consumers are biased and initially unmotivated,
       but motivated in the second period at the return decision *)
       (* These consumers initially buy provided that uunmotmot - p - x ≥
        0 where x is their distance *)
       (* We can characterize the demand of these consumers
        and associated transport costs and CW *)
       (* This group is only relevant for p < uunmotmot *)

In[]:= meddrp = FullSimplify[(1 - μ) μ xmedrp];
       medtcrp = FullSimplify[μ (1 - μ) Integrate[x, {x, 0, xmedrp}]];
       medcwrp = FullSimplify[meddrp * (uhigh - p) - medtcrp];

In[]:= (* The final group of consumers are the second-period unmotivated consumers *)
       (* Clearly, if p is low enough that they confirm the purchase,
       they also initially registered *)
       (* This group is only relevant for p < ulow *)

In[]:= lowdrp = (1 - μ) xlowrp;
       lowtcrp = FullSimplify[(1 - μ) Integrate[x, {x, 0, xlowrp}]];
       lowcwrp = FullSimplify[lowdrp * (ulow - p) - lowtcrp];
```

- Get Demands for Different Strategies

```
In[]:= (* p > uunmotmot *)

In[]:= dexclrp = FullSimplify[excldrp]
```

Out[]= $(-p + u + \Delta)\ \mu^2$

```
In[]:= (* uunmotmot > p > ulow *)

In[]:= dmedrp = FullSimplify[excldrp + meddrp]
```

Out[]= $\left(-p + u + \Delta + \alpha\, \Delta\, (-1 + \mu)\right)\ \mu$

```
In[]:= (* p < ulow *)

In[]:= dfullrp = FullSimplify[excldrp + meddrp + lowdrp]
```

Out[]= $-p + u + \Delta\ \left(1 + \alpha\ (-1 + \mu)\right)\ \mu$

- Analysis of Strategies

- Some Unmotivated do not Return

```
In[]:= (* requires p < ulow *)

In[]:= (* get solution conditional on targeting strategy *)
```

In[•]:=
```
tempsol = FullSimplify[Maximize[dfullrp * (p - c), p], assumptions]
```
Out[•]= $\left\{\frac{1}{4}\left(c-u+\Delta\mu\left(-1+\alpha-\alpha\mu\right)\right)^2,\ \left\{p\to\frac{1}{2}\left(c+u+\Delta\left(1+\alpha\left(-1+\mu\right)\right)\mu\right)\right\}\right\}$

In[•]:=
```
(* Use solution to characterize profits, price, quantity, CW *)
```
In[•]:=
```
pfullrp = p /. tempsol[[2]];
proffullrp = tempsol[[1]];
qfullrp = FullSimplify[dfullrp /. p → pfullrp];
cwfullrp = FullSimplify[exclcwrp + medcwrp + lowcwrp /. p → pfullrp, assumptions];
```
In[•]:=
```
(* Get cutoff such that optimal price interior, i.e. p ≤ ulow *)
```
In[•]:=
```
FullSimplify[Solve[pfullrp == ulow, c]]
```
Out[•]= $\left\{\left\{c\to u+\Delta\mu\left(-1+\alpha-\alpha\mu\right)\right\}\right\}$

In[•]:=
```
cfullrphigh = FullSimplify[c /. Solve[pfullrp == ulow, c][[1]]];
```

- Some Unmotivated Tender

In[•]:=
```
(* requires ulow < p < uunmotmot *)
```
In[•]:=
```
(* get solution conditional on targeting strategy *)
```
In[•]:=
```
tempsol = FullSimplify[Maximize[dmedrp * (p - c), p], assumptions]
```
Out[•]= $\left\{\frac{1}{4}\left(-c+u+\Delta+\alpha\Delta\left(-1+\mu\right)\right)^2\mu,\ \left\{p\to\frac{1}{2}\left(c+u+\Delta+\alpha\Delta\left(-1+\mu\right)\right)\right\}\right\}$

In[•]:=
```
(* Use solution to characterize profits, price, quantity, CW *)
```
In[•]:=
```
pmedrp = p /. tempsol[[2]];
profmedrp = tempsol[[1]];
qmedrp = FullSimplify[dmedrp /. {p → pmedrp}];
cwmedrp = FullSimplify[exclcwrp + medcwrp /. p → pmedrp];
```
In[•]:=
```
(* Get cost thresholds such that optimum is interior *)
```
In[•]:=
```
Solve[pmedrp == ulow, c]
```
Out[•]= $\{\{c\to u-\Delta+\alpha\Delta-\alpha\Delta\mu\}\}$

In[•]:=
```
Solve[pmedrp == uunmotmot, c]
```
Out[•]= $\{\{c\to u+\Delta-\alpha\Delta-\alpha\Delta\mu\}\}$

In[•]:=
```
cmedrplow = c /. FullSimplify[Solve[pmedrp == ulow, c][[1]]];
cmedrphigh = c /. FullSimplify[Solve[pmedrp == uunmotmot, c][[1]]];
```

- No unmotivated tender

In[•]:=
```
(* requires p > uunmotmot *)
```
In[•]:=
```
(* get solution conditional on targeting strategy *)
```
In[•]:=
```
tempsol = FullSimplify[Maximize[dexclrp * (p - c), p], assumptions]
```
Out[•]= $\left\{\frac{1}{4}\left(-c+u+\Delta\right)^2\mu^2,\ \left\{p\to\frac{1}{2}\left(c+u+\Delta\right)\right\}\right\}$

In[•]:=
```
(* Use solution to characterize profits, price, quantity, CW *)
```

```
In[•]:= pexclrp = p /. tempsol[[2]];
        profexclrp = tempsol[[1]];
        qexclrp = FullSimplify[dexclrp /. p → pexclrp];
        cwexclrp = FullSimplify[exclcwrp /. p → pexclrp];

In[•]:= (* Get cost threshold such that optimum is interior *)

In[•]:= Solve[pexclrp == uunmotmot, c]
```

Out[•]= $\{\{c \to u + \Delta - 2\,\alpha\,\Delta\}\}$

```
In[•]:= cexclrplow = c /. FullSimplify[Solve[pexclrp == uunmotmot, c][[1]]];
```

- Characterize Optimal Targeting

```
In[•]:= (* For TeX: Print Results *)

In[•]:= FullSimplify[{pfullrp, pmedrp, pexclrp}, assumptions]
```

Out[•]= $\left\{\frac{1}{2}\left(c + u + \Delta\left(1 + \alpha\left(-1 + \mu\right)\right)\mu\right), \frac{1}{2}\left(c + u + \Delta + \alpha\,\Delta\left(-1 + \mu\right)\right), \frac{1}{2}\left(c + u + \Delta\right)\right\}$

```
In[•]:= FullSimplify[{qfullrp, qmedrp, qexclrp}, assumptions]
```

Out[•]= $\left\{\frac{1}{2}\left(-c + u + \Delta\left(1 + \alpha\left(-1 + \mu\right)\right)\mu\right), \frac{1}{2}\left(-c + u + \Delta + \alpha\,\Delta\left(-1 + \mu\right)\right)\mu, \frac{1}{2}\left(-c + u + \Delta\right)\mu^2\right\}$

```
In[•]:= FullSimplify[{proffullrp, profmedrp, profexclrp}, assumptions]
```

Out[•]= $\left\{\frac{1}{4}\left(c - u + \Delta\,\mu\left(-1 + \alpha - \alpha\,\mu\right)\right)^2, \frac{1}{4}\left(-c + u + \Delta + \alpha\,\Delta\left(-1 + \mu\right)\right)^2\mu, \frac{1}{4}\left(-c + u + \Delta\right)^2\mu^2\right\}$

```
In[•]:= FullSimplify[Collect[{cwfullrp, cwmedrp, cwexclrp}, Δ], assumptions]
```

Out[•]= $\Big\{\frac{1}{8}\Big(c^2 - 2\,c\,u + u^2 + 2\left(-c + u\right)\Delta\,\mu\left(1 + \alpha - \alpha\,\mu\right) + \Delta^2\,\mu\left(4 - 3\,\mu + \alpha\left(-1 + \mu\right)\left(-2\,\mu + \alpha\left(4 + \left(-1 + \mu\right)\mu\right)\right)\right)\Big),$
$\frac{1}{8}\,\mu\left(c^2 + u^2 + 2\,u\,\Delta\left(1 + \alpha - \alpha\,\mu\right) - 2\,c\left(u + \Delta\left(1 + \alpha - \alpha\,\mu\right)\right) + \Delta^2\left(1 + \alpha\left(-1 + \mu\right)\left(-2 + \alpha\left(3 + \mu\right)\right)\right)\right),$
$\frac{1}{8}\left(-c + u + \Delta\right)^2\mu^2\Big\}$

- Comparison 1: Full to medium

```
In[•]:= FullSimplify[Solve[proffullrp == profmedrp, c], assumptions]
```

Out[•]= $\left\{\left\{c \to u + \Delta\left(1 + \alpha\left(-1 + \mu\right)\right)\sqrt{\mu}\right\}, \left\{c \to u + \Delta\sqrt{\mu}\left(-1 + \alpha - \alpha\,\mu\right)\right\}\right\}$

```
In[•]:= c1temp = c /. FullSimplify[Solve[proffullrp == profmedrp, c]][[1]];
        c2temp = c /. FullSimplify[Solve[proffullrp == profmedrp, c]][[2]];

In[•]:= FullSimplify[c1temp > cfullrphigh, assumptions]

Out[•]= True

In[•]:= FullSimplify[cfullrphigh > c2temp, assumptions]

Out[•]= True

In[•]:= (* The relevant cutoff must be the lower root,
        i.e. c2temp, which is always interior *)

In[•]:= c1rp = FullSimplify[c2temp, assumptions];
```

Comparison 2: Medium to Exclusive

In[•]:= `FullSimplify[Solve[profexclrp == profmedrp, c], assumptions]`

Out[•]= $\{\{c \to u + \Delta + \alpha \Delta (-1 + \sqrt{\mu})\}, \{c \to u - \Delta (-1 + \alpha + \alpha \sqrt{\mu})\}\}$

In[•]:=
```
c1temp = c /. FullSimplify[Solve[profexclrp == profmedrp, c]][[1]];
c2temp = c /. FullSimplify[Solve[profexclrp == profmedrp, c]][[2]];
```

In[•]:= `FullSimplify[c1temp > cmedrphigh > c2temp, assumptions]`

Out[•]= True

In[•]:= `FullSimplify[c2temp > cmedrplow, assumptions]`

Out[•]= $2 + \alpha \mu > \alpha (2 + \sqrt{\mu})$

In[•]:=
```
(* The relevant cutoff must be the lower root, i.e. c2temp,
which is not always interior (may be too low) *)
```

In[•]:= `c2rp = FullSimplify[c2temp, assumptions];`

Comparison 3: Full to Exclusive

In[•]:= `FullSimplify[Solve[profexclrp == proffullrp, c], assumptions]`

Out[•]= $\{\{c \to u - \alpha \Delta \mu\}, \{c \to u + \frac{\Delta (2 + \alpha (-1 + \mu)) \mu}{1 + \mu}\}\}$

In[•]:=
```
c1temp = c /. FullSimplify[Solve[profexclrp == proffullrp, c]][[1]];
c2temp = c /. FullSimplify[Solve[profexclrp == proffullrp, c]][[2]];
```

In[•]:= `FullSimplify[c1temp < c2temp, assumptions]`

Out[•]= True

In[•]:= `FullSimplify[c1temp > cexclrplow, assumptions]`

Out[•]= $1 + \alpha \mu < 2 \alpha$

In[•]:=
```
(* The relevant cutoff must be the lower root, i.e. c1temp,
which is not always interior (may be too low) *)
```

In[•]:= `c3rp = FullSimplify[c1temp, assumptions]`

Out[•]= $u - \alpha \Delta \mu$

Relative Location of Thresholds

In[•]:= `{c1rp, c2rp, c3rp}`

Out[•]= $\{u + \Delta \sqrt{\mu} (-1 + \alpha - \alpha \mu), u - \Delta (-1 + \alpha + \alpha \sqrt{\mu}), u - \alpha \Delta \mu\}$

In[•]:=
```
(* Strategy: Establish that Ordering is fully determined
  by comparison between α and cutoff which we call alphacritrp *)
```

In[•]:= `(* Step 1: get cutoff alphacritrp12 such that c1rp==c2rp *)`

In[•]:= `FullSimplify[Solve[c1rp == c2rp, α]]`

Out[•]= $\{\{\alpha \to \frac{1}{1 + \sqrt{\mu} - \mu}\}\}$

```
In[]:= alphacritrp12 = α /. FullSimplify[Solve[c1rp == c2rp, α]][[1]];

In[]:= FullSimplify[1 > alphacritrp12 > 0, {1 > μ > 0}]

Out[]= True

In[]:= (* alphacritrp12 is interior in (0,1), denote it as alphacritrp *)

In[]:= alphacritrp = alphacritrp12;

In[]:= (* Step 2: get cutoff alphacritrp13 such that c1rp==c3rp *)

In[]:= FullSimplify[Solve[c1rp == c3rp, α], {1 > μ > 0}]
```

Out[]= $\left\{\left\{\alpha \to \frac{1}{1+\sqrt{\mu}-\mu}\right\}\right\}$

```
In[]:= alphacritrp13 = α /. FullSimplify[Solve[c1rp == c3rp, α], {1 > μ > 0}][[1]];

In[]:= FullSimplify[alphacritrp == alphacritrp13]

Out[]= True

In[]:= (* Step 3: get cutoff alphacritrp23 such that c2rp==c3rp *)

In[]:= FullSimplify[Solve[c2rp == c3rp, α], {1 > μ > 0}]
```

Out[]= $\left\{\left\{\alpha \to \frac{1}{1+\sqrt{\mu}-\mu}\right\}\right\}$

```
In[]:= alphacritrp23 = α /. FullSimplify[Solve[c2rp == c3rp, α], {1 > μ > 0}][[1]];

In[]:= FullSimplify[alphacritrp23 == alphacritrp, {1 > μ > 0}]

Out[]= True

In[]:= (* This is the same threshold *)

In[]:= assumptionsrphighalpha = {1 > α > alphacritrp, 1 > μ > 0, u > 0, Δ > 0};

In[]:= assumptionsrplowalpha = {alphacritrp > α > 0, 1 > μ > 0, u > 0, Δ > 0};

In[]:= FullSimplify[c1rp > c3rp > c2rp, {assumptionsrphighalpha}]

Out[]= True

In[]:= FullSimplify[c1rp < c3rp < c2rp, assumptionsrplowalpha]

Out[]= True

In[]:= (* check that pmedrp is indeed interior whenever
        intermediate targeting occurs (α < alphacritrp, c ∈ (c1rp,c2rp) *)

In[]:= (* pmedrp > ulow for c = c1rp *)

In[]:= FullSimplify[pmedrp > ulow /. c → c1rp, assumptionsrplowalpha]

Out[]= True

In[]:= (* pmedrp < uunmotmot for c = c2rp *)

In[]:= FullSimplify[pmedrp < uunmotmot /. c → c2rp, assumptionsrplowalpha]

Out[]= True
```

In[∘]:=
```
(* overall we have c3rp > c1rp if and only if α < alphacritrp *)
(* So for low α c1rp and c2rp determine
 targeting behavior and intermediate targeting is possible,
for high α it is not and c3rp determines targeting behavior *)
```

- α uniquely determines targeting behavior
- Characterization Complete

In[∘]:=
```
(* What can we say about possible combinations *)
```

- 1.) Comparison CO vs BL

In[∘]:=
```
FullSimplify[ctildeco > ctilde, assumptions]
```

Out[∘]= True

In[∘]:=
```
(* cooling off period leads to weakly less exclusive targeting behavior *)
```

- Possible Cases with CO

In[∘]:=
```
(* 1.) Cooling Off *)
```

In[∘]:=
```
(* For each of the possible combinations,
utilize what we know about c relative to thresholds
 to assess directional change in quantities, price, and CW *)
```

In[∘]:=
```
(* A: Full to Full *)
```

In[∘]:=
```
FullSimplify[{Reduce[qfullco - qfull < 0], Reduce[pfullco - pfull < 0]}, assumptions]
```

Out[∘]= {True, True}

In[∘]:=
```
(* Consumer Welfare *)
```

In[∘]:=
```
cond1 =
 Simplify[Reduce[Simplify[Reduce[cwfullco > cwfull], assumptions]], (u | α | Δ | μ) ∈ ℝ]
```

Out[∘]= $c < u + \Delta\,\mu - \frac{1}{2}\,\alpha\,\Delta\,\left(-4 + 7\,\mu + \mu^2\right)$

In[∘]:=
```
FullSimplify[
 Equivalent[cond1, Simplify[Reduce[cwfullco > cwfull], assumptions]], assumptions]
```

Out[∘]= True

In[∘]:=
```
FullSimplify[cond1[[2]] > ctildeco, assumptions]
```

Out[∘]= True

In[∘]:=
```
(* Upper bound on c such that Δc > 0 is above ctildeco;
c < ctildeco is required for full targeting in baseline *)
```

In[∘]:=
```
(* This implies that ΔCW > 0 for Full → Full *)
```

In[∘]:=
```
(* B: Excl to Full *)
```

In[∘]:=
```
(* First establish Δq > 0 and Δp < 0 *)
```

In[∘]:=
```
FullSimplify[
 Reduce[FullSimplify[Reduce[{qfullco - qexcl > 0 && pfullco - pexcl < 0}], assumptions]],
 c < ctildeco]
```

Out[∘]= True

```
In[•]:= (* Consumer Welfare -- ΔCW *)

In[•]:= (* We establish that ΔCW>0 in two steps *)

In[•]:= (* For small μ it is straightforward to establish that cwfullco > cwexcl *)

In[•]:= (* Define critical μ *)

In[•]:= mucrit = 2 Sqrt[13] - 7;

In[•]:= FullSimplify[Simplify[
          Reduce[cwfullco - cwexcl > 0 && 0 < μ < mucrit && assumptions], μ < mucrit], assumptions]

Out[•]= True

In[•]:= (* For large μ this becomes slightly more involved *)

In[•]:= Reduce[cwfullco > cwexcl && 1 > μ > mucrit && assumptions]
```

Out[•]= $-7+2\sqrt{13}<\mu<1 \;\&\&\; 0<\alpha<1 \;\&\&\; \Delta>0 \;\&\&\; u>0 \;\&\&\; \left(c<u+\Delta\mu+2\alpha\Delta\mu-\sqrt{-3\alpha^2\Delta^2\mu+14\alpha^2\Delta^2\mu^2+\alpha^2\Delta^2\mu^3} \;||\; c>u+\Delta\mu+2\alpha\Delta\mu+\sqrt{-3\alpha^2\Delta^2\mu+14\alpha^2\Delta^2\mu^2+\alpha^2\Delta^2\mu^3}\right)$

```
In[•]:= temp = Simplify[Simplify[
          Reduce[cwfullco > cwexcl && 1 > μ > mucrit && assumptions], μ > mucrit], assumptions]
```

Out[•]= $c<u+\Delta\left(\mu+2\alpha\mu-\alpha\sqrt{\mu\left(-3+14\mu+\mu^2\right)}\right) \;||\; c>u+\Delta\left(\mu+2\alpha\mu+\alpha\sqrt{\mu\left(-3+14\mu+\mu^2\right)}\right)$

```
In[•]:= (* We focus on the first inequality and denote by ctemp
          the upper bound on c which ensures that cwfullco > cwexcl *)

In[•]:= ctemp = temp[[1]][[2]]
```

Out[•]= $u+\Delta\left(\mu+2\alpha\mu-\alpha\sqrt{\mu\left(-3+14\mu+\mu^2\right)}\right)$

```
In[•]:= (* If c < ctemp and μ > mucrit we have ΔCW > 0 *)

In[•]:= Simplify[Simplify[Simplify[Reduce[cwfullco > cwexcl && assumptions],
          {c < ctemp, μ > mucrit}], assumptions], c < ctemp]

Out[•]= True

In[•]:= (* As full targeting materializes post policy adoption we know that c < ctildeco *)

In[•]:= (* It hence suffices for ΔCW > 0 to show that ctildeco < ctemp *)

In[•]:= Simplify[Simplify[Reduce[ctildeco < ctemp], μ > mucrit], assumptions]

Out[•]= True

In[•]:= (* Overall we hence have ΔCW > 0 for Excl → Full *)

In[•]:= (* C: Excl to Excl *)

In[•]:= (* First establish Δq < 0 and Δp = 0 *)

In[•]:= FullSimplify[Reduce[qexclco - qexcl < 0 && pexclco - pexcl == 0],
         assumptions && c > ctildeco && c < expmot]

Out[•]= True

In[•]:= (* ΔCW is ambiguous *)
```

In[ ]:=
```
t = Simplify[Simplify[Reduce[cwexclco - cwexcl > 0 && assumptions], assumptions],
   {c > ctildeco, c < expmot}]
```
Out[ ]= $c > u + \Delta\,(3\,\alpha\,(-1+\mu)+\mu)$

In[ ]:=
```
(* Define the critical c as ctemp *)
```

In[ ]:=
```
ctemp = t[[2]]
```
Out[ ]= $u + \Delta\,(3\,\alpha\,(-1+\mu)+\mu)$

In[ ]:=
```
Simplify[ctildeco < ctemp < expmot, assumptions]
```
Out[ ]= $5\,\mu > 3$

In[ ]:=
```
(* For μ > 3/5, there are constellations where Excl → Excl reduces welfare,
specifically for c > ctemp with ctemp ∈ (ctildeco,expmot) *)
```

- 2.) Comparison BL to RP

In[ ]:=
```
FullSimplify[c1rp < ctilde, {1 > μ > 0, Δ > 0, 0 < α < alphacritrp}]
```
Out[ ]= $1+\alpha\,\sqrt{\mu} > 2\,\alpha$

In[ ]:=
```
FullSimplify[c2rp > ctilde, {1 > μ > 0, Δ > 0, 1 > α > 0}]
```
Out[ ]= True

In[ ]:=
```
FullSimplify[c3rp > ctilde, {1 > μ > 0, Δ > 0, 1 > α > alphacritrp}]
```
Out[ ]= True

In[ ]:=
```
(* Know that Exclusive targeting materializes for larger cutoff due to c2rp >
 ctilde and exclusive targeting iff c > Max[c2rp,c3rp] *)
```

In[ ]:=
```
(* So Full → Excl is ruled out *)
```

In[ ]:=
```
(* Get Δp, Δq,
ΔCW for the 5 possible combinations of pre and post policy targeting behavior *)
```

In[ ]:=
```
(* A: Full to Full *)
```

In[ ]:=
```
(* Establish Δq < 0, Δp < 0 *)
```

In[ ]:=
```
FullSimplify[qfullrp - qfull < 0 && pfullrp - pfull < 0, assumptions]
```
Out[ ]= True

In[ ]:=
```
(* Establish ΔCW > 0 *)
```

In[ ]:=
```
(* Get cutoff ctemp such that this holds for c < ctemp *)
```

In[ ]:=
```
t = FullSimplify[Reduce[cwfullrp > cwfull && assumptions], assumptions]
```
Out[ ]= $c < u + \frac{2\,\Delta}{\alpha} + \frac{1}{2}\,\Delta\,\mu\,(2+\alpha-\alpha\,\mu)$

In[ ]:=
```
ctemp = t[[2]]
```
Out[ ]= $u + \frac{2\,\Delta}{\alpha} + \frac{1}{2}\,\Delta\,\mu\,(2+\alpha-\alpha\,\mu)$

In[ ]:=
```
(* ctemp > ctilde is sufficient for ΔCW > 0 as c < ctilde <
 ctemp follows due to full market coverage pre intervention *)
```

In[•]:= FullSimplify[ctemp > ctilde, assumptions]

Out[•]= True

In[•]:= (* B: Full to Int *)
(* Exploit that Intermediate targeting with RP only possible for α < alphacritrp as this is necessary for c1rp < c3rp < c2rp *)

In[•]:= (* Full to Int requires that c1rp < c < c2rp but c < ctilde, so ctilde > c1rp is necessary *)

(* This implies a tighter bound on α, namely 0 < α < fullmedalpha < alphacritrp *)

In[•]:= FullSimplify[Reduce[c1rp < ctilde && assumptions], assumptions]

Out[•]= $2\alpha \leq 1 \,||\, \alpha\,(4 + \alpha\,(-4 + \mu)) > 1$

(* we can ignore the α ≤ 1/2 as the constraint derived below subsumes this *)

In[•]:= Simplify[Solve[c1rp == ctilde, α], assumptions]

Out[•]= $\left\{\left\{\alpha \to \frac{1}{2 - \sqrt{\mu}}\right\}\right\}$

In[•]:= fullmedalpha = α /. Simplify[Solve[c1rp == ctilde, α], assumptions][[1]]

Out[•]= $\frac{1}{2 - \sqrt{\mu}}$

In[•]:= Simplify[0 < 1/2 < fullmedalpha < alphacritrp, 1 > μ > 0]

Out[•]= True

In[•]:= (* fullmedalpha > 1/2 ensures that this is indeed the more relevant constraint *)

In[•]:= FullSimplify[ctilde > c1rp && assumptions && fullmedalpha > α, assumptions && fullmedalpha > α]

Out[•]= True

In[•]:= (* Having established this, we now consider Δp, Δq and ΔCW *)

In[•]:= (* First establish Δp > 0 *)

In[•]:= FullSimplify[pmedrp > pfull, assumptions]

Out[•]= True

In[•]:= (* Next consider Δq *)

In[•]:= t = Simplify[Reduce[qmedrp < qfull && assumptions], assumptions]

Out[•]= $c < u + \alpha\,\Delta\,\mu$

In[•]:= ctemp = t[[2]]

Out[•]= $u + \alpha\,\Delta\,\mu$

In[•]:= (* qmedrp < qfull for c < ctemp *)

In[•]:= condtemp = FullSimplify[Reduce[ctilde > ctemp > c1rp], assumptions && 0 < α < fullmedalpha]

Out[•]= $\mu > \frac{\alpha^2}{(1 - 2\alpha)^2}$

```
In[∘]:= FullSimplify[Exists[α, 0 < α < fullmedalpha && assumptions, condtemp], assumptions]
Out[∘]= True

(* Clearly the above can be satisfied so that Δq > 0 and Δq <
 0 is possible for c < ctemp and c > ctemp, respectively; Δq is hence ambiguous *)

In[∘]:= (* Next consider ΔCW *)

In[∘]:= cond = FullSimplify[Reduce[cwmedrp > cwfull && assumptions], assumptions]
```

Out[∘]= $u < c + (1 + \alpha)\, \Delta\, \sqrt{\mu} + \alpha\, \Delta\, \mu \,\&\&\, c + \alpha\, \Delta\, \mu < u + (1 + \alpha)\, \Delta\, \sqrt{\mu}$

```
In[∘]:= (* Define c1temp, c2temp such that ΔCW > 0 iff c ∈ (c1temp,c2temp) *)

In[∘]:= t = FullSimplify[Solve[cwmedrp == cwfull, c], assumptions]
```

Out[∘]= $\{\{c \to u - (1 + \alpha)\, \Delta\, \sqrt{\mu} - \alpha\, \Delta\, \mu\},\ \{c \to u + (1 + \alpha)\, \Delta\, \sqrt{\mu} - \alpha\, \Delta\, \mu\}\}$

```
In[∘]:= c1temp = c /. t[[1]]; c2temp = c /. t[[2]];

In[∘]:= FullSimplify[c1temp < c2temp, assumptions]
Out[∘]= True

In[∘]:= FullSimplify[Equivalent[cond, c1temp < c && c < c2temp], assumptions]
Out[∘]= True

In[∘]:= (* Recall that we know that c ∈
 (c1rp,ctilde) for this targeting behavior combination *)

In[∘]:= (* It is hence sufficient to establish c1rp >
 c1temp and ctilde < c2temp for ΔCW > 0 *)

In[∘]:= FullSimplify[c1rp > c1temp && ctilde < c2temp, assumptions]
Out[∘]= True

In[∘]:= (* This implies ΔCW > 0 *)

In[∘]:= (* D: Excl to Full *)

In[∘]:= (* For this we require that c > ctilde and c < Min[c1rp,c3rp] *)

In[∘]:= (* First establish Δq > 0 *)

In[∘]:= Simplify[Reduce[qfullrp > qexcl && assumptions], assumptions]
```

Out[∘]= $c + (-1 + 2\, \alpha)\, \Delta\, \mu < u$

```
In[∘]:= (* define critical ctemp such that Δq > 0 iff c < ctemp *)

In[∘]:= t = Solve[qfullrp == qexcl, c]
```

Out[∘]= $\{\{c \to u + \Delta\, \mu - 2\, \alpha\, \Delta\, \mu\}\}$

```
In[∘]:= ctemp = c /. t[[1]]
```

Out[∘]= $u + \Delta\, \mu - 2\, \alpha\, \Delta\, \mu$

```
In[∘]:= (* c < ctemp is sufficient for Δq > 0. In turn,
ctemp > c3rp is sufficient for this as c <
 c3rp is necessary for full targeting post intervention *)
```

```
In[•]:= FullSimplify[ctemp > c3rp, assumptions]
Out[•]= True

In[•]:= (* We hence have Δq > 0 *)

In[•]:= (* Next establish Δp < 0 *)

In[•]:= FullSimplify[pfullrp < pexcl, assumptions]
Out[•]= True

In[•]:= (* Next establish ΔCW > 0 *)

In[•]:= (* We establish ΔCW > 0 by (i) showing that ΔCW > 0 at c → c1rp and c → c3rp,
        and (ii) that ΔCW is decreasing in c for c < c1rp and c < c3rp *)

In[•]:= (* This implies that irrespective of whether c1rp or c3rp is the threshold
         for full targeting post policy adoption, ΔCW is strictly positive as c ∈
          (ctilde,Min[c1rp,c3rp]) is required for Excl → Full *)

In[•]:= (* Define ΔCW = deltaCW *)

In[•]:= deltaCW = FullSimplify[cwfullrp - cwexcl, assumptions]
```

Out[•]= $-\frac{1}{8}\left(-1+\mu\right)\left(c^2+u^2+2\,u\left(1+2\,\alpha\right)\Delta\mu-2\,c\left(u+\left(1+2\,\alpha\right)\Delta\mu\right)+\Delta^2\mu\left(4+\mu+4\,\alpha\,\mu-\alpha^2\left(1+\mu\right)^2\right)\right)$

```
In[•]:= (* Show that ΔCW > 0 at c→c1rp *)

In[•]:= FullSimplify[FullSimplify[deltaCW /. c → c1rp, assumptions] > 0, assumptions]
Out[•]= True

In[•]:= (* Show that ΔCW > 0 at c→c3rp *)

In[•]:= FullSimplify[FullSimplify[deltaCW /. c → c3rp, assumptions] > 0, assumptions]
Out[•]= True

        (* Get threshold ctemp such that ΔCW is decreasing in c iff c <
         ctemp: D[ΔCW,c] < 0 for c < ctemp *)

In[•]:= t = FullSimplify[Reduce[D[deltaCW, c] < 0 && assumptions], assumptions]
```

Out[•]= $c < u+\Delta\mu+2\,\alpha\,\Delta\mu$

```
In[•]:= ctemp = t[[2]]
```

Out[•]= $u+\Delta\mu+2\,\alpha\,\Delta\mu$

```
In[•]:= FullSimplify[D[deltaCW, c] < 0, {c < ctemp, 1 > μ > 0, 1 > α > 0, u > 0}]
Out[•]= True

In[•]:= (* Show that ctemp > c1rp and ctemp > c3rp *)

In[•]:= FullSimplify[ctemp > c1rp && ctemp > c3rp, assumptions]
Out[•]= True

In[•]:= (* Overall we hence have ΔCW > 0 because deltaCW is decreasing in c up to c1rp
          or c3rp which are the respective upper bound for the targeting region *)

In[•]:= (* H: Excl to Med *)
```

```
In[•]:= (* Note that Excl → Int implies multiple things *)

In[•]:= (* First, it needs to be that c1rp < c3rp < c2rp, i.e. α < alphacritrp,
        so that Intermediate targeting post policy can materialize *)

In[•]:= (* Second, for Excl → Int we require that c ∈ (c1rp,c2rp) and c > ctilde *)

In[•]:= (* As ctilde < c2rp holds generically,
        there always is such a parameter constellation provided that α < alphacritrp *)

In[•]:= (* Note that alphacritrp > 1/2 *)

In[•]:= FullSimplify[alphacritrp > 1 / 2, 1 > μ > 0]

Out[•]= True

In[•]:= (* On Δq: We show Δq > 0 iff α < 1/2;
        α can lie above or below 1/2 as it is only required to be below alphacritrp *)

In[•]:= Simplify[Reduce[qmedrp > qexcl && assumptions], assumptions]

Out[•]= 2 α < 1

In[•]:= (* On Δp: We show Δp > 0 iff α < 1/2;
        α can lie above or below 1/2 as it is only required to be below alphacritrp *)

In[•]:= Simplify[Reduce[pmedrp > pexcl && assumptions], assumptions]

Out[•]= 2 α < 1

In[•]:= (* Finally we need to consider ΔCW *)

In[•]:= deltaCW = FullSimplify[cwmedrp - cwexcl, assumptions]
```

$$\text{Out[•]= } \frac{1}{8} \Delta \left(-1+\mu\right) \mu \left(\left(1+2\alpha\right) \left(2c-2u-\Delta\right)+\left(-1-2\alpha+4\alpha^2\right) \Delta \mu\right)$$

```
In[•]:= (* Define ctemp such that ΔCW > 0 iff c < ctemp *)

In[•]:= t = Simplify[Reduce[deltaCW > 0 && assumptions],
          assumptions && c > ctilde && c < c2rp && 0 < α < alphacritrp]
```

$$\text{Out[•]= } c < u+\frac{\Delta \left(1+\mu-4\alpha^2\mu+2\alpha\left(1+\mu\right)\right)}{2+4\alpha}$$

```
In[•]:= ctemp = t[[2]]
```

$$\text{Out[•]= } u+\frac{\Delta \left(1+\mu-4\alpha^2\mu+2\alpha\left(1+\mu\right)\right)}{2+4\alpha}$$

```
In[•]:= (* ctemp lies strictly above ctilde, so ΔCW > 0 for c → ctilde *)

In[•]:= Simplify[Reduce[ctemp > ctilde], assumptions]

Out[•]= True

In[•]:= Simplify[Reduce[(deltaCW /. c → ctilde) > 0 && assumptions], assumptions]

Out[•]= True

In[•]:= (* If we could also establish that ctemp > c2rp then by c <
         c2rp for Intermediate targeting post adoption we know that c <
         ctemp and hence ΔCW > 0 *)
```

In[∘]:=
```
(* In contrast, if ctemp < c2rp then there exists a cost range,
specifically (ctemp,c2rp), such that ΔCW < 0 *)
```

In[∘]:=
```
(* define deltac = ctemp - c2rp *)
```

In[∘]:=
```
deltac = FullSimplify[ctemp - c2rp, assumptions]
```

Out[∘]= $$\Delta\left(-1+\alpha+\alpha\sqrt{\mu}+\frac{1+\mu+2\alpha\left(1+\mu-2\alpha\mu\right)}{2+4\alpha}\right)$$

In[∘]:=
```
Simplify[Reduce[deltac < 0 && assumptions && 0 < α < alphacritrp],
 assumptions && 0 < α < alphacritrp]
```

Out[∘]= $$2\alpha<1\ \&\&\ 2\sqrt{\frac{\alpha^2\left(1+2\alpha\right)^3\left(1-7\alpha^2+10\alpha^3\right)}{\left(1+2\alpha-4\alpha^2\right)^4}}+\mu<\frac{1+2\alpha-6\alpha^2+24\alpha^4}{\left(1+2\alpha-4\alpha^2\right)^2}$$

In[∘]:=
```
(* For α > 1/2, we hence have deltac > 0 and therefore ctemp >
 c2rp which implies that c < c2rp < ctemp and thus ΔCW > 0 *)
```

In[∘]:=
```
FullSimplify[Reduce[ctemp > c2rp && assumptions && 1/2 < α],
 assumptions && 1/2 < α < alphacritrp]
```

Out[∘]= True

In[∘]:=
```
FullSimplify[Reduce[deltaCW > 0 && c < ctemp && assumptions], assumptions && c < ctemp]
```

Out[∘]= True

In[∘]:=
```
(* Restrict attention to the case where α < 1/2 *)
```

In[∘]:=
```
t = FullSimplify[Solve[FullSimplify[Reduce[deltac == 0], assumptions], α], assumptions]
```

Out[∘]= $$\left\{\left\{\alpha\to\frac{\sqrt{\mu}+\mu-\sqrt{\left(1+\sqrt{\mu}\right)\left(4-7\mu+5\mu^{3/2}\right)}}{4\left(-1-\sqrt{\mu}+\mu\right)}\right\}\right\}$$

In[∘]:=
```
exclintcritalpha = α /. t[[1]]
```

Out[∘]= $$\frac{\sqrt{\mu}+\mu-\sqrt{\left(1+\sqrt{\mu}\right)\left(4-7\mu+5\mu^{3/2}\right)}}{4\left(-1-\sqrt{\mu}+\mu\right)}$$

In[∘]:=
```
FullSimplify[alphacritrp > exclintcritalpha > 0, 1 > μ > 0]
```

Out[∘]= True

In[∘]:=
```
FullSimplify[exclintcritalpha < 1/2, 1 > μ > 0]
```

Out[∘]= True

```
(* For α < exclintcritalpha there exists a cost range c ∈
 (ctemp,c2rp) such that Excl → Int targeting materializes with ΔCW < 0 *)
```

In[∘]:=
```
FullSimplify[c1rp < ctemp && c1rp < ctilde && ctilde < ctemp && ctemp < c2rp,
 {0 < α < exclintcritalpha, 1 > μ > 0, Δ > 0}]
```

Out[∘]= True

```
(* Verify that cwexcl >
 0 indeed holds (this is necessary as CW > 0 always holds with a RP) *)
```

```
In[•]:= FullSimplify[cwexcl > 0 /. c → ctemp, assumptions && 0 < α < exclintcritalpha]
Out[•]= True

In[•]:= (* For α > exclintcritalpha, ΔCW > 0 holds generically for Excl → Int *)

In[•]:= (* As exclintcritalpha < 1/2,
        this subsumes α > 1/2 which we had already established *)

In[•]:= (* I: Excl to Excl *)

In[•]:= (* First adress Δq *)

In[•]:= (* Define ctemp such that Δq > 0 iff c > ctemp *)

In[•]:= t = Simplify[Reduce[qexclrp > qexcl, c], assumptions]
Out[•]= c > u + α Δ

In[•]:= ctemp = t[[2]]
Out[•]= u + α Δ

In[•]:= (* For Excl →
         Excl we require that c > Max[c2rp,c3rp] as c2rp > ctilde holds generically *)

In[•]:= FullSimplify[ctemp < c2rp || ctemp < c3rp, assumptions]
```

$$\text{Out}[\circ]= \alpha\left(2+\sqrt{\mu}\right) < 1$$

```
In[•]:= (* For small α we hence have ctemp < Max[c2rp,c3rp] and thus that Δq > 0 *)

In[•]:= (* For large α in contrast Δq < 0 is possible *)

In[•]:= (* Overall, Δq is ambiguous *)

In[•]:= (* Next show Δp > 0 *)

In[•]:= Simplify[Reduce[pexclrp > pexcl && assumptions], assumptions]
Out[•]= True

In[•]:= (* Finally analyze ΔCW *)

In[•]:= cond1 = Simplify[Reduce[cwexclrp > cwexcl, c], assumptions]
```

$$\text{Out}[\circ]= u < c + \Delta\left(\alpha + \sqrt{\alpha^2\left(4 - 3\mu\right) + \mu + 2\alpha\mu}\right) \,\&\&\, c + \alpha\Delta < u + \Delta\sqrt{\alpha^2\left(4 - 3\mu\right) + \mu + 2\alpha\mu}$$

```
In[•]:= (* Above shows that ΔCW > 0 iff c ∈
         (c1temp,c2temp) with c1temp and c2temp characterized as follows *)

In[•]:= t = FullSimplify[Solve[cwexclrp == cwexcl, c], assumptions]
```

$$\text{Out}[\circ]= \left\{\left\{c \to u + \Delta\left(-\alpha + \sqrt{\mu + \alpha\left(4\alpha + 2\mu - 3\alpha\mu\right)}\right)\right\}, \left\{c \to u - \Delta\left(\alpha + \sqrt{\mu + \alpha\left(4\alpha + 2\mu - 3\alpha\mu\right)}\right)\right\}\right\}$$

```
In[•]:= c1temp = c /. t[[1]]; c2temp = c /. t[[2]];

In[•]:= FullSimplify[c1temp > c2temp, assumptions]
Out[•]= True

        (* Verify that condition on cost is equivalent to ΔCW > 0 *)

In[•]:= cond = Simplify[Reduce[cwexclrp > cwexcl && assumptions], assumptions];
```

```
In[•]:= FullSimplify[
         Reduce[Equivalent[cond, c2temp < c && c < c1temp] && assumptions], assumptions]

Out[•]= True

        (* Note that c1temp > expmot and hence that for ΔCW >
         0 it is sufficient that c2temp < c2rp as c >
         c2rp is necessary for Exclusive Targeting post intervention *)

In[•]:= FullSimplify[c1temp > expmot, assumptions]

Out[•]= True

In[•]:= FullSimplify[c2temp < ctilde, assumptions]

Out[•]= True

In[•]:= (* Hence, we necessarily have ΔCW > 0 *)
```