```
ClearAll["Global`*"]

(* Initialize by defining utilities, uhigh and ulow,
and state-contingent predicted utilities, uhighlow and ulowhigh *)

(*)Utility of motivated consumer in consumption state

uhigh = u + Δ;
(*)Utility of unmotivated consumer in consumption state

ulow = u;
(*) Predicted utility of unmotivated consumer for motivated future state

uhighlow = ulow + (1 - α) Δ;
(*) Predicted utility of motivated consumer for unmotivated future state

ulowhigh = ulow + α Δ;

(* Get predicted utilities when currently motivated and unmotivated, respectively *)

(* Persistence in Motivation states is given by ρ ∈ [0,1) *)
(*)Predicted expected future utility of currently motivated consumer

umot = (μ (1 - ρ) + ρ) uhigh + (1 - ρ) (1 - μ) ulowhigh;
(*)Predicted expected future utility of currently unmotivated consumer

uunmot = μ (1 - ρ) uhighlow + (ρ + (1 - ρ) (1 - μ)) ulow;

(* Create Lists of assumptions for ease of reference *)

asslist = {u > 0, Δ > 0, 1 > α > 0, 1 > μ > 0, 1 > ρ >= 0};

assand = u > 0 && Δ > 0 && 1 > α > 0 && 1 > μ > 0 && 1 > ρ ≥ 0;

(* Define expected utility exput and expected
 utility given persistence in motivation state *)

(* Expected utility *)


exput = μ uhigh + (1 - μ) ulow;
(* Actual expected future utility of motivated consumer *)


expmot = FullSimplify[(ρ + (1 - ρ) μ) uhigh + (1 - ρ) (1 - μ) ulow];
(* Actual expected future utility of motivated consumer *)

expunmot = FullSimplify[(1 - ρ) μ uhigh + (ρ + (1 - ρ) (1 - μ)) ulow];

(* Print umot and uunmot for comparison *)

FullSimplify[{umot, uunmot}]
```

$$\{u + \Delta\,(\mu + \alpha\,(-1+\mu)\,(-1+\rho) + \rho - \mu\,\rho),\ u + (-1+\alpha)\,\Delta\,\mu\,(-1+\rho)\}$$

■ Baseline

```
(* Strategy 1: exclusive targeting,
cater only to motivated consumers by charging p =
 pexcl equal to the predicted utility of motivated consumers umot *)
(*Demand and price *)
```

```
dexcl = μ; pexcl = umot; profexcl = FullSimplify[dexcl * (pexcl - c), asslist]
```

$$\mu \left(-c + u + \Delta \left(\alpha + \mu - \alpha \mu + \left(-1 + \alpha\right) \left(-1 + \mu\right) \rho\right)\right)$$

```
(* Compute Consumer Welfare *)

cwexcl = FullSimplify[dexcl * (expmot - pexcl), asslist];

(* Strategy 2: full targeting, cater to all consumers by charging p =
 pfull equal to the predicted utility of unmotivated consumers uunmot *)
(* Demand and price *)

dfull = 1;
pfull = uunmot;
proffull = FullSimplify[dfull * (pfull - c), asslist]
```

$$-c + u + \left(-1 + \alpha\right) \Delta \mu \left(-1 + \rho\right)$$

```
cwfull = FullSimplify[dfull * (μ expmot + (1 - μ) expunmot - pfull), asslist];

(* Obtain cost threshold such that exclusive targeting preferred iff c > ctilde *)

temp = Simplify[Reduce[profexcl > proffull && assand, c, Reals], asslist]
```

$$c > u + \frac{\Delta \mu \left(-1 + \mu + \alpha \left(-2 + \mu\right) \left(-1 + \rho\right) + 2 \rho - \mu \rho\right)}{-1 + \mu}$$

```
ctilde = temp[[2]]
```

$$u + \frac{\Delta \mu \left(-1 + \mu + \alpha \left(-2 + \mu\right) \left(-1 + \rho\right) + 2 \rho - \mu \rho\right)}{-1 + \mu}$$

■ Cooling Off

```
(* Strategy 1: exclusive targeting,
cater only to motivated consumers by charging p =
 pexclco equal to the predicted utility of motivated consumers umot *)
(* Reflect that purchase only occurs if confirmed by
 again motivated consumers in second period *)

dexclco = FullSimplify[μ * (ρ + (1 - ρ) μ)];
pexclco = umot;
profexclco = FullSimplify[dexclco * (pexclco - c), asslist]
```

$$\mu \left(\mu + \rho - \mu \rho\right) \left(-c + u + \Delta \left(\alpha + \mu - \alpha \mu + \left(-1 + \alpha\right) \left(-1 + \mu\right) \rho\right)\right)$$

```
(* Get Welfare *)

cwexclco = FullSimplify[dexclco * (expmot - pexclco), asslist];

(* Strategy 2: full targeting, cater to all consumers by charging p =
 pfullco equal to the predicted utility of unmotivated consumers *)

dfullco = 1;
pfullco = uunmot;
proffullco = FullSimplify[dfullco * (pfullco - c), asslist]
```

$$-c + u + \left(-1 + \alpha\right) \Delta \mu \left(-1 + \rho\right)$$

```
(* Get Welfare *)

cwfullco = FullSimplify[dfullco * (μ expmot + (1 - μ) expunmot - pfullco), asslist];
```

```
(* Obtain cost threshold such that exclusive targeting preferred iff c > ctildeco *)

temp = FullSimplify[Reduce[profexclco > proffullco && assand, c, Reals], asslist]
```

$$c > u + \frac{\Delta\,\mu\,\left(1-\rho+\alpha\,\left(-1+\rho\right)\,\left(1+\mu-\mu^2+\left(-1+\mu\right)^2\,\rho\right)-\left(\mu+\rho-\mu\,\rho\right)^2\right)}{\left(-1+\mu\right)\,\left(-1+\mu\,\left(-1+\rho\right)\right)}$$

```
ctildeco = temp[[2]]
```

$$u + \frac{\Delta\,\mu\,\left(1-\rho+\alpha\,\left(-1+\rho\right)\,\left(1+\mu-\mu^2+\left(-1+\mu\right)^2\,\rho\right)-\left(\mu+\rho-\mu\,\rho\right)^2\right)}{\left(-1+\mu\right)\,\left(-1+\mu\,\left(-1+\rho\right)\right)}$$

■ Verify Behavior for Cooling Off

```
(* Cutoff Increases so only weakly less
 exclusive targeting is possible post intervention *)

FullSimplify[ctildeco > ctilde, asslist]
```

```
True
```

```
(* Case 1: Full Coverage Pre- & Post *)

FullSimplify[{dfullco == dfull, pfullco == pfull, cwfullco == cwfull}, asslist]
```

```
{True, True, True}
```

```
(* Case 2: Exclusive Coverage Pre, Full Coverage Post *)
(* Higher demand, lower price, higher CW *)

FullSimplify[{dfullco > dexcl, pfullco < pexcl, cwfullco > cwexcl}, asslist]
```

```
{True, True, True}
```

```
(* Case 3: Exclusive Coverage Pre & Post *)
(* lower demand than before, same price, higher CW *)

FullSimplify[{dexclco < dexcl, pexclco == pexcl, cwexclco > cwexcl}, asslist]
```

```
{True, True, True}
```

■ Return Policy

```
(* Strategy 1: Exclusive targeting,
cater only to initially motivated consumers who consume in the motivated state *)

dexclrp = μ (ρ + (1 - ρ) μ);
pexclrp = uhigh;
profexclrp = FullSimplify[dexclrp * (pexclrp - c)]
```

$$(-c+u+\Delta)\;\mu\;(\mu+\rho-\mu\,\rho)$$

```
cwexclrp = FullSimplify[dexclrp * (uhigh - pexclrp), asslist];

(* Strategy 2: Intermediate targeting,
cater only to second period motivated consumers by charging p =
 pmedrp equal to predicted utility in motivated state by unmotivated consumers *)
```

```
dmedrp = FullSimplify[μ (ρ + (1 - ρ) μ) + (1 - μ) (1 - ρ) μ];
pmedrp = uhighlow;
profmedrp = FullSimplify[dmedrp * (pmedrp - c)]
```

$$(-c + u + \Delta - \alpha \Delta)\ \mu$$

```
cwmedrp = FullSimplify[dmedrp * (uhigh - pmedrp), asslist];

(* Strategy 3: Full targeting, cater to all consumers by
 charging price equal to consumption utility in unmotivated state *)

dfullrp = 1;
pfullrp = ulow;
proffullrp = FullSimplify[dfullrp * (pfullrp - c)]
```

$$-c + u$$

```
cwfullrp = FullSimplify[exput - pfullrp];

(* Thresholds from pairwise comparison of strategies *)

(* Threshold 1: Medium targeting preferred to full targeting iff c > c1rp *)

Simplify[Reduce[profmedrp > proffullrp && assand, c, Reals], asslist]
```

$$c + \frac{(-1 + \alpha)\ \Delta\ \mu}{-1 + \mu} > u$$

```
temp = FullSimplify[Solve[profmedrp == proffullrp, c], asslist]
```

$$\left\{\left\{c \to u - \frac{(-1 + \alpha)\ \Delta\ \mu}{-1 + \mu}\right\}\right\}$$

```
c1rp = c /. temp[[1]]
```

$$u - \frac{(-1 + \alpha)\ \Delta\ \mu}{-1 + \mu}$$

```
(* Threshold 2:
 Exclusive targeting preferred to intermediate targeting iff c > c2rp *)

Simplify[Reduce[profexclrp > profmedrp && assand, c, Reals], asslist]
```

$$c + \frac{\Delta\ (-1 + \alpha + \mu + \rho - \mu\ \rho)}{(-1 + \mu)\ (-1 + \rho)} > u$$

```
temp = FullSimplify[Solve[profexclrp == profmedrp, c], asslist]
```

$$\left\{\left\{c \to u + \Delta - \frac{\alpha\ \Delta}{(-1 + \mu)\ (-1 + \rho)}\right\}\right\}$$

```
c2rp = c /. temp[[1]]
```

$$u + \Delta - \frac{\alpha\ \Delta}{(-1 + \mu)\ (-1 + \rho)}$$

```
(* Threshold 3: Exclusive targeting preferred to full targeting iff c > c3rp *)
Simplify[Reduce[profexclrp > proffullrp && assand, c, Reals], asslist]
```

$$c > u + \frac{\Delta\ \mu\ (\mu\ (-1 + \rho) - \rho)}{(-1 + \mu)\ (-1 + \mu\ (-1 + \rho))}$$

```
temp = FullSimplify[Solve[profexclrp == proffullrp, c], asslist]
```

$$\left\{\left\{c \to u + \Delta - \frac{\Delta}{\left(-1+\mu\right)\ \left(-1+\mu\ \left(-1+\rho\right)\right)}\right\}\right\}$$

```
c3rp = c /. temp[[1]]
```

$$u + \Delta - \frac{\Delta}{\left(-1+\mu\right)\ \left(-1+\mu\ \left(-1+\rho\right)\right)}$$

```
(* Reduce Cases *)
```

```
Reduce[c1rp > c3rp > c2rp && assand]
```

$$u > 0 \&\& 0 \le \rho < 1 \&\& 0 < \mu < 1 \&\& \frac{-1+\rho}{-1-\mu+\mu\,\rho} < \alpha < 1 \&\& \Delta > 0$$

```
alphacritrp = (1 - ρ) / (1 + μ - μ ρ);
```

```
asslisttemp = {u > 0, Δ > 0, 1 > α > alphacritrp, 1 > μ > 0, 1 > ρ ≥ 0}
```

$$\left\{u > 0,\ \Delta > 0,\ 1 > \alpha > \frac{1-\rho}{1+\mu-\mu\,\rho},\ 1 > \mu > 0,\ 1 > \rho \ge 0\right\}$$

```
FullSimplify[c1rp > c3rp > c2rp, asslisttemp]
```

True

```
asslisttemp = {u > 0, Δ > 0, 0 < α < alphacritrp, 1 > μ > 0, 1 > ρ ≥ 0}
```

$$\left\{u > 0,\ \Delta > 0,\ 0 < \alpha < \frac{1-\rho}{1+\mu-\mu\,\rho},\ 1 > \mu > 0,\ 1 > \rho \ge 0\right\}$$

```
FullSimplify[c1rp < c3rp < c2rp, asslisttemp]
```

True

```
(* Depends only on Critical α → for low α,
c1rp < c3rp < c2rp so that c1rp, c2rp determines targeting *)
```

```
(* Depends only on Critical α → for high α,
c1rp > c3rp > c2rp so that c3rp determines targeting *)
```

■ Verify Behavior for Return Policy

```
(* Additional Case relative to Baseline: Full →
 Exclusive Targeting is possible for large ρ *)
```

```
(* To illustrate this consider α = alphacritrp which implies c1rp = c2rp = c3rp *)
```

```
FullSimplify[c1rp == c2rp == c3rp, α == alphacritrp]
```

True

```
diff = FullSimplify[ctilde - c1rp /. α → alphacritrp]
```

$$-\frac{\Delta\,\mu\ \left(1+\mu\ \left(-1+\rho\right)-2\,\rho\right)\ \left(-1+\rho\right)}{-1+\mu\ \left(-1+\rho\right)}$$

```
Simplify[Reduce[diff > 0, ρ], {Δ > 0, 1 > μ > 0, ρ < 1}]
```

$$1 + \mu\,\rho < \mu + 2\,\rho$$

```
(* Clearly this is satisfied for ρ >
  (1-μ)/(2-μ) in which case it is possible that c <
  ctilde (Full pre) but c > c1rp = c2rp = c3rp (Excl post) *)
```

- Behavior of Δp, Δq, ΔCW

```
(* Full Pre & Post *)
```

```
FullSimplify[{dfullrp == dfull, pfullrp < pfull, cwfullrp > cwfull}, asslist]
```

```
{True, True, True}
```

```
(* Full Pre & Intermediate Post *)
```

```
FullSimplify[{dmedrp < dfull, pmedrp > pfull, cwmedrp < cwfull}, asslist]
```

```
{True, True, ρ > 0}
```

```
(* Note change: ΔCW < 0 instead of ΔCW =
 0 without persistence -- Identification issue remains, market size required *)
```

```
(* Full Pre & Exclusive Post *)
```

```
FullSimplify[{dexclrp < dfull, pexclrp > pfull, cwexclrp < cwfull}, asslist]
```

```
{True, True, True}
```

```
(* Excl Pre & Full Post *)
```

```
FullSimplify[{dfullrp > dexcl, pfullrp < pexcl, cwfullrp > cwexcl}, asslist]
```

```
{True, True, True}
```

```
(* Excl Pre & Intermediate Post *)
```

```
FullSimplify[{dmedrp == dexcl, pmedrp < pexcl, cwmedrp > cwexcl}, asslist]
```

$$\left\{\text{True},\ 1+\mu\,\rho+\alpha\,(\mu+\rho)<\mu+\rho+\alpha\,\left(2+\mu\,\rho\right),\ \text{True}\right\}$$

```
(* Excl Pre & Post *)
```

```
FullSimplify[{dexclrp < dexcl, pexclrp > pexcl, cwexclrp > cwexcl}, asslist]
```

```
{True, True, True}
```