```
ClearAll["Global`*"]
```

- Preliminaries
- We allow for arbitrary $\mu$, $\alpha$, and fraction $\rho$ of rational consumers
- Throughout, we operate with interior parameter values $\mu$, $\alpha$, $\rho \in (0,1)$. We set $\delta = 1$ for ease of exposition.
- Including the boundary cases is straightforward, as is behavior for c equal to thresholds.

```
ulow = u; uhigh = u + Δ;

(* Expected Utility *)

exput = ulow + μ (uhigh - ulow)
```

$u + \Delta\,\mu$

```
(* Predicted Expected Utility of Motivated biased Consumers *)

expmot = FullSimplify[μ uhigh + (1 - μ) (α uhigh + (1 - α) ulow)]
```

$u + \Delta\,(\alpha + \mu - \alpha\,\mu)$

```
(* Predicted Expected Utility of Unmotivated biased Consumers *)

expunmot = FullSimplify[μ (α ulow + (1 - α) uhigh) + (1 - μ) ulow]
```

$u - (-1 + \alpha)\,\Delta\,\mu$

```
assumptions = {1 > α > 0, 1 > μ > 0, u > 0, Δ > 0, 1 > ρ > 0}
```

$\{1 > \alpha > 0,\ 1 > \mu > 0,\ u > 0,\ \Delta > 0,\ 1 > \rho > 0\}$

- Baseline -- Three targeting strategies

```
(* Throughout, q/p/prof/cw denote quantity, prices, profits, and Consumer Welfare *)

(* Full/Med/Excl denote the three different targeting strategies,
the interpretation of which varies across policy settings *)

(* Strategy 1: Cater to full market,
pricing based on predicted expected utility of biased unmotivated *)

qfull = 1;
pfull = expunmot;
proffull = (pfull - c) * qfull;
cwfull = qfull (exput - pfull);

(* Strategy 2: Intermediate targeting,
pricing based on rational consumers and hence true expected utility *)

qmed = ρ + μ (1 - ρ);
pmed = exput;
profmed = (pmed - c) * qmed;
cwmed = qmed (exput - pmed);

(* Strategy 3: Exclusive targeting,
pricing based on predicted expected utility of biased motivated *)
```

```
qexcl = μ (1 - ρ);
pexcl = expmot;
profexcl = (pexcl - c) * qexcl;
cwexcl = qexcl * (exput - pexcl);
```

```
(* Check that above characterizations are correct *)
```

```
FullSimplify[{qfull, pfull, proffull, cwfull}, assumptions]
```

$\{1, u - (-1+\alpha)\,\Delta\,\mu, -c + u - (-1+\alpha)\,\Delta\,\mu, \alpha\,\Delta\,\mu\}$

```
FullSimplify[{qmed, pmed, profmed, cwmed}, assumptions]
```

$\{\mu + \rho - \mu\,\rho, u + \Delta\,\mu, (-c + u + \Delta\,\mu)\,(\mu + \rho - \mu\,\rho), 0\}$

```
FullSimplify[{qexcl, pexcl, profexcl, cwexcl}, assumptions]
```

$\{\mu - \mu\,\rho, u + \Delta\,(\alpha + \mu - \alpha\,\mu), \mu\,(-c + u + \Delta\,(\alpha + \mu - \alpha\,\mu))\,(1-\rho), -\alpha\,\Delta\,(-1+\mu)\,\mu\,(-1+\rho)\}$

- Solution of Baseline Model

```
(* Step A: Get cost thresholds such that
   firm is indifferent between two targeting strategies *)
```

```
(* c1: between full and med; c2: between med and excl; c3: between full and excl *)
```

```
FullSimplify[Solve[proffull == profmed, c], assumptions]
```

$$\left\{\left\{c \to u + \Delta\,\mu\left(1 - \frac{\alpha}{(-1+\mu)\,(-1+\rho)}\right)\right\}\right\}$$

```
c1 = FullSimplify[c /. Solve[proffull == profmed, c][[1]], assumptions];
```

```
FullSimplify[Solve[profmed == profexcl, c], assumptions]
```

$$\left\{\left\{c \to u + \frac{\Delta\,\mu\,(\rho + \alpha\,(-1 + \mu + \rho - \mu\,\rho))}{\rho}\right\}\right\}$$

```
c2 = FullSimplify[c /. Solve[profmed == profexcl, c][[1]], assumptions];
```

```
FullSimplify[Solve[proffull == profexcl, c], assumptions]
```

$$\left\{\left\{c \to u + \Delta\left(\mu + \alpha\left(1 - \mu + \frac{1}{-1 + \mu - \mu\,\rho}\right)\right)\right\}\right\}$$

```
c3 = FullSimplify[c /. Solve[proffull == profexcl, c][[1]], assumptions];
```

```
(* Can be established that unique cutoff in ρ exists
 which determines ordering of thresholds (verification below) *)
```

```
rhotilde = ρ /. FullSimplify[Solve[c1 == c2, ρ][[1]]]
```

$$\frac{3 + 2\,(-2+\mu)\,\mu - \sqrt{5 + 4\,(-2+\mu)\,\mu}}{2\,(-1+\mu)^2}$$

```
(* Check that rhotilde is interior *)
```

```
FullSimplify[1 > rhotilde > 0, 1 > μ > 0]
```

```
True
```

- Use $\rho$ relative to cutoff to obtain ordering and potential strategies

- Case 1 : Rho large = > lots of rational consumers = > c1 < c3 < c2

```
assumptions_rhohigh = {1 > α > 0, 1 > μ > 0, u > 0, Δ > 0, 1 > ρ > rhotilde};

FullSimplify[c1 < c3 < c2, {assumptions_rhohigh}]
True

(* In this case: For c < c1 Full targeting;
c ∈ (c1,c2) Medium Targeting, c > c2 Exclusive Targeting *)

(* Logic is identical to that of Return Policy
 thresholds in Baseline Model with only one consumer type *)
```

- Case 2: Rho small => few rational consumers => c1 > c3 > c2

```
assumptions_rholow = {1 > α > 0, 1 > μ > 0, u > 0, Δ > 0, 0 < ρ < rhotilde};

FullSimplify[c1 > c3 > c2, assumptions_rholow]
True

(* In this case: c < c3 Full Targeting,
c > c3 Exclusive Targeting; never rational targeting *)

(* Overall: Can define ctildelow = Min{c1,c3} and ctildehigh =
  Max{c2,c3} to have pricing determined by these two thresholds *)
```

- Cooling Off: Three Candidates

```
(* Same as before, first define strategies which carry over *)

(* co at end of variables denotes cooling off *)

qfullco = 1;
pfullco = expunmot;
proffullco = (pfullco - c) * qfullco;
cwfullco = qfullco (exput - pfullco);

qmedco = ρ + (1 - ρ) μ^2;
pmedco = exput;
profmedco = (pmedco - c) * qmedco;
cwmedco = (exput - pmedco) * qmedco;

qexclco = (1 - ρ) μ^2;
pexclco = expmot;
profexclco = (pexclco - c) * qexclco;
cwexclco = qexclco * (exput - pexclco);

(* Check that above characterizations are correct *)

FullSimplify[{qfullco, pfullco, proffullco, cwfullco}, assumptions]
{1, u - (-1 + α) Δ μ, -c + u - (-1 + α) Δ μ, α Δ μ}

FullSimplify[{qmedco, pmedco, profmedco, cwmedco}, assumptions]
{-μ^2 (-1 + ρ) + ρ, u + Δ μ, (-c + u + Δ μ) (-μ^2 (-1 + ρ) + ρ), 0}
```

```
FullSimplify[{qexclco, pexclco, profexclco, cwexclco}, assumptions]
```

$\{-\mu^2 (-1+\rho),\ u+\Delta (\alpha+\mu-\alpha \mu),\ \mu^2 (-c+u+\Delta (\alpha+\mu-\alpha \mu)) (1-\rho),\ -\alpha \Delta (-1+\mu) \mu^2 (-1+\rho)\}$

```
(* Get cutoffs as before; co at end of variable denotes cooling off *)

FullSimplify[Solve[proffullco == profmedco, c], assumptions]
```

$\left\{\left\{c \to u+\Delta \mu \left(1-\frac{\alpha}{(-1+\mu^2) (-1+\rho)}\right)\right\}\right\}$

```
c1co = FullSimplify[c /. Solve[proffullco == profmedco, c][[1]]];

FullSimplify[Solve[profmedco == profexclco, c], assumptions]
```

$\left\{\left\{c \to u+\frac{\Delta \mu (\rho+\alpha \mu (-1+\mu+\rho-\mu \rho))}{\rho}\right\}\right\}$

```
c2co = FullSimplify[c /. Solve[profmedco == profexclco, c][[1]]];

FullSimplify[Solve[proffullco == profexclco, c], assumptions]
```

$\left\{\left\{c \to u+\Delta \left(\alpha+\mu-\alpha \mu-\frac{\alpha}{1+\mu^2 (-1+\rho)}\right)\right\}\right\}$

```
c3co = FullSimplify[c /. Solve[proffullco == profexclco, c][[1]]];

(* Determined ρ threshold which determines ordering *)

rhotildeco = ρ /. FullSimplify[Solve[c1co == c2co, ρ][[1]]]
```

$\frac{1+2 (-1+\mu)^2 \mu (1+\mu)-\sqrt{1+4 (-1+\mu)^2 \mu (1+\mu)}}{2 (-1+\mu)^2 \mu (1+\mu)}$

```
(* Check that threshold is interior *)

FullSimplify[1 > rhotildeco > 0, 1 > μ > 0]
```

True

$\text{assumptions}_{\text{rhohighco}}$ = {1 > α > 0, 1 > μ > 0, u > 0, Δ > 0, 1 > ρ > rhotildeco};

$\text{assumptions}_{\text{rholowco}}$ = {1 > α > 0, 1 > μ > 0, u > 0, Δ > 0, 0 < ρ < rhotildeco};

- Case 1: Rho large = > lots of rational consumers = > c1co < c3co < c2co

FullSimplify[c1co < c3co < c2co, $\text{assumptions}_{\text{rhohighco}}$]

True

```
(* In this case: For c < c1co Full targeting;
c ∈ (c1co,c2co) Medium Targeting, c > c2co Exclusive Targeting *)
```

- Case 2: Rho small => few rational consumers => c1co > c3co > c2co

FullSimplify[c1co > c3co > c2co, $\text{assumptions}_{\text{rholowco}}$]

True

```
(* In this case: c < c3co Full Targeting,
c > c3co Exclusive Targeting; never rational targeting *)

(* Analysis follows RP in Baseline Model with only one consumer type *)
```

- Impact of CO

```
(* First establish relative ordering of rho cutoffs *)

FullSimplify[Reduce[rhotilde > rhotildeco], 1 > μ > 0]
```

$1 + 2\mu < \sqrt{5}$

```
(* Ordering is ambiguous and depends on
 fraction of motivated consumers in a given period *)

(* Characterize this cutoffmu so that rhotilde > rhotildeco iff μ < cutoffmu *)

cutoffmu = μ /. Solve[rhotilde == rhotildeco, μ][[2]]
```

$\frac{1}{2}\left(-1+\sqrt{5}\right)$

```
(* Check that cutoffmu is interior *)

FullSimplify[1 > cutoffmu > 0]

True

{FullSimplify[rhotilde > rhotildeco, 0 < μ < cutoffmu],
 FullSimplify[rhotilde < rhotildeco, 1 > μ > cutoffmu]}

{True, True}
```

- Towards assessing the impact: First get generic comparisons given targeting behavior pre & post policy introduction

```
(* 1.) Cooling Off *)

(* A: Full to Full *)

FullSimplify[
 {qfullco - qfull == 0, pfullco - pfull == 0, cwfullco - cwfull == 0}, assumptions]

{True, True, True}

(* B: Full to Med *)

FullSimplify[{qmedco - qfull < 0, pmedco - pfull > 0, cwmedco - cwfull < 0}, assumptions]

{True, True, True}

(* C: Full to Excl *)

FullSimplify[{qexclco - qfull < 0, pexclco - pfull > 0, cwexclco - cwfull < 0}, assumptions]

{True, True, True}

(* D: Med to Full *)

FullSimplify[{qfullco - qmed > 0, pfullco - pmed < 0, cwfullco - cwmed > 0}, assumptions]

{True, True, True}

(* E: Med to Med *)

FullSimplify[{qmedco - qmed < 0, pmedco - pmed == 0, cwmedco - cwmed == 0}, assumptions]

{True, True, True}
```

```
(* F: Med to Excl *)

FullSimplify[{qexclco - qmed < 0, pexclco - pmed > 0, cwexclco - cwmed < 0}, assumptions]
{True, True, True}

(* G: Excl to Full *)

FullSimplify[{qfullco - qexcl > 0, pfullco - pexcl < 0, cwfullco - cwexcl > 0}, assumptions]
{True, True, True}

(* H: Excl to Med *)
(* Exploit that we know that ρ >
 rhotildeco is necessary so that medium targeting can materialize post policy *)

FullSimplify[
 {Reduce[qmedco - qexcl > 0], pmedco - pexcl < 0, cwmedco - cwexcl > 0}, assumptions_rhohighco]
{ρ + μ (μ + ρ) > μ (1 + μ ρ), True, True}

(* Δq cannot be signed *)

(* Cutoff ρ such that Δq switches signs lies above rhotildeco *)

FullSimplify[Solve[ρ + μ (μ + ρ) == μ (1 + μ ρ), ρ][[1]][[1]][[2]] > rhotildeco, 1 > μ > 0]
True

(* I: Excl to Excl *)

FullSimplify[{qexclco - qexcl < 0, pexclco - pexcl == 0, cwexclco - cwexcl > 0}, assumptions]
{True, True, True}
```

- Generic Cost Threshold Comparison

```
FullSimplify[c3co > c3, assumptions]
True

FullSimplify[c2co > c2, assumptions]
True

FullSimplify[c1co > c1, assumptions]
True

(* Targeting Behavior can never become more exclusive with cooling off period *)
```

- Targeting behavior uniquely determines price, quantity and CW movement except for Exclusive -> Med in which case quantity movement is ambiguous

```
(* only need to assess which targeting behavior
  combination can materialize and check Excl→Med if it is relevant *)
```

- Case 1: $\mu$ sufficiently small ($\mu$<cutoffmu) then rhotildeco < rhotilde
- Case 1.1: rho very small ($\rho$<rhotildeco): c3, c3co determine targeting

```
FullSimplify[c3co > c3, {1 > α > 0, rhotildeco > ρ > 0, 0 < μ < cutoffmu, Δ > 0}]
True
```

```
(* c < c3: Full in both, c ∈ (c3,c3co): Exclusive Pre,
Full Post, c > c3co: Exclusive Pre and Post *)
```

- Case 1.2: rho small, not very small (rhotildeco<$\rho$<rhotilde), c3 and c1co,c2co determine targeting

```
FullSimplify[c3 < c1co, {1 > α > 0, rhotildeco < ρ < rhotilde, 0 < μ < cutoffmu, Δ > 0}]
True

FullSimplify[c1co < c2co, {1 > α > 0, rhotildeco < ρ < rhotilde, 0 < μ < cutoffmu, Δ > 0}]
True

(* c < c3: Full in Both, c ∈ (c3,c1co): Exclusive Pre, Full Post,
c ∈ (c1co,c2co): Exclusive Pre, Medium Post, c > c2co: Exclusive Pre and Post *)

(* Check whether Δq can be signed in case Excl→Med *)

(* For ρ -> rhotildeco, Δq < 0 *)

FullSimplify[Reduce[qexcl > qmedco] /. ρ → rhotildeco, {0 < μ < cutoffmu}]
True

(* For ρ → rhotilde, μ sufficiently small: Δq > 0 *)

FullSimplify[FullSimplify[Reduce[qexcl < qmedco] /. {ρ → rhotilde}] /. μ → 0.01]
True

(* Both cases are possible; sign of Δq is ambiguous *)
```

- Case 1.3: rho large ($\rho$>rhotilde), c1,c2 and c1co,c2co determine targeting

```
FullSimplify[c1 < c1co < c2co, {1 > α > 0, 1 > ρ > rhotilde, 0 < μ < cutoffmu, u > 0, Δ > 0}]
True

FullSimplify[c2 < c2co, {1 > α > 0, 1 > ρ > rhotilde, 0 < μ < cutoffmu, u > 0, Δ > 0}]
True

FullSimplify[c1co < c2, {1 > α > 0, 1 > ρ > rhotilde, 0 < μ < cutoffmu, u > 0, Δ > 0}]
```

$$1 + \mu^3 \left(-1 + \rho\right)^2 + \rho^2 < \mu \left(1 + \mu\right) \left(-1 + \rho\right)^2 + 3 \rho$$

```
(* Relative position of c1co to c2 ambiguous *)

(* Two possible Scenarios *)

(* Case 1.3a -- c1co < c2 (rho large): c < c1: Full in Both,
c ∈ (c1,c1co): Medium Pre, Full Post, c ∈ (c1co,c2): Medium Pre & Post,
c ∈ (c2,c2co): Exclusive Pre, Medium Post, c > c2co: Exclusive Pre and Post *)

(* Case 1.3b -- c1co > c2 (rho not too large): c < c1: Full in Both,
c ∈ (c1,c2): Medium Pre, Full Post, c ∈ (c2,c1co): Exclusive Pre, Full Post,
c ∈ (c1co,c2co): Exclusive Pre, Medium Post, c > c2co: Exclusive Pre and Post *)

(* Check whether quantity can be signed in case Excl→Med*)
```

```
FullSimplify[qexcl > qmedco, {ρ > rhotilde, 0 < μ < cutoffmu, u > 0, Δ > 0}]
```

$(-1+\mu)\,\mu\,(-1+\rho) > \rho$

```
Reduce[qexcl > qmedco /. ρ → rhotilde /. μ → cutoffmu]
```

True

```
Reduce[qexcl < qmedco /. ρ → rhotilde /. μ → 0.01]
```

True

```
(* Δq ambiguous *)

(* Both cases are possible; sign of Δq is ambiguous *)
```

- Case 2 : $\mu$ large = > different cutoff ordering (rhotildeco > rhotilde)
- Case 2.1 : rho very small ($\rho$ < rhotilde) : c3, c3co determine targeting

```
FullSimplify[c3co > c3, {1 > α > 0, 1 > ρ > 0, 1 > μ > cutoffmu, u > 0, Δ > 0}]
```

True

```
(* c < c3: Full in both, c ∈ (c3,c3co): Exclusive Pre,
Full Post, c > c3co: Exclusive Pre and Post *)
```

- Case 2.2 : rho small, not very small (rhotilde < $\rho$ < rhotildeco), c3co and c1, c2 determine targeting

```
FullSimplify[c3co > c2,
 {1 > α > 0, rhotilde < ρ < rhotildeco, 1 > μ > cutoffmu, u > 0, Δ > 0, u > 0, Δ > 0}]
```

True

```
FullSimplify[c2 > c1,
 {1 > α > 0, rhotilde < ρ < rhotildeco, 1 > μ > cutoffmu, u > 0, Δ > 0, u > 0, Δ > 0}]
```

True

```
(* c < c1: Full in Both, c ∈ (c1,c2): Medium Pre, Full Post,
c ∈ (c2,c3co): Excl Pre, Full Post, c > c3co: Excl Pre and Post *)
```

- Case 2.3 : rho large ($\rho$ > rhotildeco), c1, c2 and c1co, c2co determine targeting

```
FullSimplify[c1 < c1co < c2co, {1 > α > 0, 1 > ρ > rhotildeco, 1 > μ > cutoffmu, u > 0, Δ > 0}]
```

True

```
FullSimplify[c2co > c2, {1 > α > 0, 1 > ρ > rhotildeco, 1 > μ > cutoffmu, u > 0, Δ > 0}]
```

True

```
FullSimplify[c1co > c2, {1 > α > 0, 1 > ρ > rhotildeco, 1 > μ > cutoffmu, u > 0, Δ > 0}]
```

$1+\mu^3\,(-1+\rho)^2+\rho^2 > \mu\,(1+\mu)\,(-1+\rho)^2+3\,\rho$

```
(* c2 can lie either above or below c1co *)

(* Case 2.3a -- c1co < c2 (rho large): c < c1: Full in Both,
c ∈ (c1,c1co): Medium Pre, Full Post, c ∈ (c1co,c2): Medium Pre & Post,
c ∈ (c2,c2co): Exclusive Pre, Medium Post, c > c2co: Exclusive Pre and Post *)
```

```
(* Case 2.3b -- c1co > c2 (rho not too large): c < c1: Full in Both,
c ∈ (c1,c2): Medium Pre, Full Post, c ∈ (c2,c1co): Exclusive Pre, Full Post,
c ∈ (c1co,c2co): Exclusive Pre, Medium Post, c > c2co: Exclusive Pre and Post *)

(* Check whether quantity can be signed in case Excl→Med*)

FullSimplify[qexcl > qmedco, {1 > ρ > rhotildeco, 1 > μ > cutoffmu}]
(-1 + μ) μ (-1 + ρ) > ρ

Reduce[qexcl > qmedco /. ρ → rhotildeco /. μ → cutoffmu]
True

Reduce[qexcl > qmedco /. ρ → 0.99 /. μ → cutoffmu]
False

(* Same as before *)
```

- Overall: All possible combinations with weakly more inclusive post-intervention targeting behavior are possible; signs of Δq, Δp, ΔCW are determined by targeting except for Excl->Med

- 

- Return Policy

```
(* Again three targeting strategies,
identical to baseline (presence of rational consumers only changes quantities) *)

(* Relevant prices: motivated utility,
unmotivated prediction for motivated, unmotivated utility *)

(* Initialize unmotivated biased consumers' prediction for motivated state *)

uunmotmot = FullSimplify[α ulow + (1 - α) uhigh];

(* Initialize Prices/Quantities/Profits/Surplus for three strategies *)

qfullrp = 1;
pfullrp = ulow;
proffullrp = (pfullrp - c) * qfullrp;
cwfullrp = qfullrp * (exput - pfullrp);

qmedrp = μ;
pmedrp = uunmotmot;
profmedrp = (pmedrp - c) * qmedrp;
cwmedrp = qmedrp * (uhigh - pmedrp);

qexclrp = μ (ρ + μ (1 - ρ));
pexclrp = uhigh;
profexclrp = (pexclrp - c) * qexclrp;
cwexclrp = qexclrp * (uhigh - pexclrp);
```

```
(* Check that above characterizations are correct *)
FullSimplify[{qfullrp, pfullrp, proffullrp, cwfullrp}, assumptions]
FullSimplify[{qmedrp, pmedrp, profmedrp, cwmedrp}, assumptions]
FullSimplify[{qexclrp, pexclrp, profexclrp, cwexclrp}, assumptions]
```

$\{1, u, -c+u, \Delta\mu\}$

$\{\mu, u+\Delta-\alpha\Delta, (-c+u+\Delta-\alpha\Delta)\mu, \alpha\Delta\mu\}$

$\{\mu(\mu+\rho-\mu\rho), u+\Delta, (-c+u+\Delta)\mu(\mu+\rho-\mu\rho), 0\}$

```
(* Get thresholds as before, rp denotes Return Policy *)
FullSimplify[Solve[proffullrp == profmedrp, c], assumptions]
```

$$\left\{\left\{c \to u - \frac{(-1+\alpha)\Delta\mu}{-1+\mu}\right\}\right\}$$

```
c1rp = FullSimplify[c /. Solve[proffullrp == profmedrp, c][[1]], assumptions];
FullSimplify[Solve[profmedrp == profexclrp, c], assumptions]
```

$$\left\{\left\{c \to u + \Delta - \frac{\alpha\Delta}{(-1+\mu)(-1+\rho)}\right\}\right\}$$

```
c2rp = FullSimplify[c /. Solve[profmedrp == profexclrp, c][[1]], assumptions];
FullSimplify[Solve[proffullrp == profexclrp, c], assumptions]
```

$$\left\{\left\{c \to u + \Delta - \frac{\Delta}{(-1+\mu)(-1+\mu(-1+\rho))}\right\}\right\}$$

```
c3rp = FullSimplify[c /. Solve[proffullrp == profexclrp, c][[1]], assumptions];
(* Characterize ρ cutoff which determines threshold cost ordering *)
rhotilderp = ρ /. Solve[c1rp == c2rp, ρ][[1]]
```

$$\frac{-1+\alpha+\alpha\mu}{-1+\alpha\mu}$$

```
FullSimplify[1 > rhotilderp, {1 > α > 0, 1 > μ > 0}]
```

True

```
FullSimplify[rhotilderp < 0, {1 > α > 0, 1 > μ > 0}]
```

$\alpha+\alpha\mu > 1$

```
(* Possible that rhotilderp < 0 in which case ρ < rhotilderp not possible *)
(* Store restriction that possible as extra assumption *)
lowrhorppossible = α + α μ < 1
```

$\alpha+\alpha\mu < 1$

- Case 1: $\rho$ < rhotilderp => c1rp < c3rp < c2rp, c1rp and c2rp determine targeting

```
FullSimplify[c1rp < c3rp < c2rp, {1 > α > 0, ρ < rhotilderp, 1 > μ > 0, u > 0, Δ > 0}]
```

True

```
(* In this case: For c < c1rp Full targeting;
c ∈ (c1rp,c2rp) Medium Targeting, c > c2rp Exclusive Targeting *)
```

- Case 2: $\rho$ > rhotilderp => c1rp > c3rp > c2rp, c3rp determines targeting

```
FullSimplify[c1rp > c3rp > c2rp, {1 > α > 0, 1 > ρ > rhotilderp, 1 > μ > 0, u > 0, Δ > 0}]
True

(* In this case: c < c3rp Full Targeting,
c > c3rp Exclusive Targeting; never medium targeting *)
```

- Towards assessing the impact: First get generic comparisons given targeting behavior pre & post policy introduction

```
(* 2.) Return Policy *)

(* A: Full to Full *)

FullSimplify[{qfullrp - qfull == 0, pfullrp - pfull < 0, cwfullrp - cwfull > 0}, assumptions]
{True, True, True}

(* B: Full to Med *)

FullSimplify[{qmedrp - qfull < 0, pmedrp - pfull > 0, cwmedrp - cwfull == 0}, assumptions]
{True, True, True}

(* C: Full to Excl *)

FullSimplify[{qexclrp - qfull < 0, pexclrp - pfull > 0, cwexclrp - cwfull < 0}, assumptions]
{True, True, True}

(* D: Med to Full *)

FullSimplify[{qfullrp - qmed > 0, pfullrp - pmed < 0, cwfullrp - cwmed > 0}, assumptions]
{True, True, True}

(* E: Med to Med, exploit that lowrhorppossible has to hold *)

FullSimplify[FullSimplify[{qmedrp - qmed < 0, pmedrp - pmed > 0, cwmedrp - cwmed > 0},
  assumptions], {lowrhorppossible, 1 > α > 0, 1 > μ > 0}]
{True, α + μ < 1, True}

(* Δp ambiguous *)

(* F: Med to Excl *)

FullSimplify[{qexclrp - qmed < 0, pexclrp - pmed > 0, cwexclrp - cwmed == 0}, assumptions]
{True, True, True}

(* G: Excl to Full *)

FullSimplify[{qfullrp - qexcl > 0, pfullrp - pexcl < 0, cwfullrp - cwexcl > 0}, assumptions]
{True, True, True}

(* H: Excl to Med *)
```

```
FullSimplify[
 FullSimplify[{qmedrp - qexcl > 0, Reduce[pmedrp - pexcl > 0], cwmedrp - cwexcl > 0},
  assumptions], {lowrhorppossible, 1 > α > 0, 1 > μ > 0}]
```

$\{\text{True}, \mu < 1 + \alpha\,(-2 + \mu), \text{True}\}$

```
(* Δp ambiguous *)

(* I: Excl to Excl *)

FullSimplify[
 {Reduce[qexclrp - qexcl < 0], pexclrp - pexcl > 0, cwexclrp - cwexcl > 0}, assumptions]
```

$\{\mu + 2\,\rho < 1 + \mu\,\rho, \text{True}, \text{True}\}$

```
(* Δq ambiguous *)

(* Δp, Δq and ΔCW generically signed except for Quantity when Excl→
 Excl and Price when Med/Excl→Med *)
```

- Impact of RP

```
(* Obtain ordering of ρ-threshodls *)

FullSimplify[rhotilde > rhotilderp, {1 > α > 0, 1 > μ > 0}]
```

$1 + \alpha\left(2 + \mu\left(-5 + 2\,\mu + \sqrt{5 + 4\,(-2 + \mu)\,\mu}\right)\right) > \sqrt{5 + 4\,(-2 + \mu)\,\mu}$

```
(* No generic ordering possible *)

(* Get critical α so that α relative to
 threshold alphatilde determined relative ordering *)

alphatilde = α /. Solve[rhotilde == rhotilderp, α][[1]]
```

$$\frac{-1 + \sqrt{5 - 8\,\mu + 4\,\mu^2}}{2 - 5\,\mu + 2\,\mu^2 + \mu\,\sqrt{5 - 8\,\mu + 4\,\mu^2}}$$

```
(* Check that interior *)

FullSimplify[1 > alphatilde > 0, 1 > μ > 0]
```

True

```
(* Check that α relative to alphatilde determines ordering *)

FullSimplify[rhotilde > rhotilderp, {1 > α > alphatilde, 1 > μ > 0}]
```

True

```
FullSimplify[rhotilde < rhotilderp, 0 < α < alphatilde]
```

True

- Now characterize Impact for different cases on $\alpha$ and $\rho$
- Case 1: $\alpha$ < alphatilde which implies rhotilde < rhotilderp

```
(* 1a: ρ < rhotilde < rhotilderp. c3 determines pricing in baseline,
c1rp and c2rp in return policy *)

FullSimplify[c3 > c1rp, {0 < α < alphatilde, 0 < ρ < rhotilde, 1 > μ > 0, Δ > 0}]
```

True

```
FullSimplify[c3 > c2rp,
 {0 < α < alphatilde, 0 < ρ < rhotilde < rhotilderp, 1 > μ > 0, Δ > 0, u > 0}]
```

$$\Delta\left(-1+\mu+\alpha\left(1-\mu+\frac{1}{(-1+\mu)\,(-1+\rho)}+\frac{1}{-1+\mu-\mu\,\rho}\right)\right)>0$$

```
(* Two possible scenarios *)

(* 1.a.i:  c3 > c2rp > c1rp. c < c1rp: Full in Both,
c ∈ (c1rp,c2rp): Full Pre, Medium Post, c ∈ (c1rp,c3): Full Pre,
Exclusive Post, c > c3: Exclusive Pre and Post *)

(* 1.a.ii: c2rp > c3 > c1rp. c < c1rp: Full in Both,
c ∈ (c1rp,c3): Full Pre, Medium Post, c ∈ (c3,c2rp): Exclusive Pre,
Medium Post, c > c2rp: Exclusive Pre and Post *)

(* Check whether quantity comparison given for Excl → Excl *)

FullSimplify[qexcl > qexclrp, {alphatilde > α > 0, 0 < ρ < rhotilde, 1 > μ > 0}]
```

True

```
(* Check whether price comparison given for Excl → Med *)

FullSimplify[Reduce[pexcl < pmedrp /. α → alphatilde],
 {0 < ρ < rhotilde, 1 > μ > 0, u > 0, Δ > 0}]
```

False

```
FullSimplify[Reduce[pexcl < pmedrp /. α → 0], {0 < ρ < rhotilde, 1 > μ > 0, u > 0, Δ > 0}]
```

True

```
(* Δp ambiguous *)

(* *)

(* 1b: rhotilderp > ρ > rhotilde. c1 and c2 determine pricing in baseline,
c1rp and c2rp in return policy *)

FullSimplify[c2 > c1 > c1rp,
 {0 < α < alphatilde, rhotilde < ρ < rhotilderp, 1 > μ > 0, u > 0, Δ > 0}]
```

True

```
FullSimplify[c2rp > c1,
 {0 < α < alphatilde, rhotilde < ρ < rhotilderp, 1 > μ > 0, u > 0, Δ > 0}]
```

$$1+\mu\,\rho>\alpha+\mu+\rho$$

```
FullSimplify[c2rp > c2,
 {0 < α < alphatilde, rhotilde < ρ < rhotilderp, 1 > μ > 0, u > 0, Δ > 0}]
```

$$\alpha\left((-1+\mu)^2\,\mu\,(-1+\rho)^2-\rho\right)>(-1+\mu)^2\,(-1+\rho)\,\rho$$

```
(* Three possible scenarios *)

(* 1.b.i:    c2rp > c2 > c1 > c1rp. c < c1rp: Full in Both,
c ∈ (c1rp,c1): Full Pre, Medium Post, c ∈ (c1,c2): Medium Pre and Post,
c ∈ (c2,c2rp): Exclusive Pre, Medium Post, c > c2rp: exclusive Pre and Post *)
```

```
(* 1.b.ii:   c2 > c2rp > c1 > c1rp. c < c1rp: Full in Both,
c ∈ (c1rp,c1): Full Pre, Medium Post, c ∈ (c1,c2rp): Medium Pre and Post,
c ∈ (c2rp,c2): Medium Pre, Exclusive Post, c > c2: exclusive Pre and Post *)

(* 1.b.iii:  c2 > c1 > c2rp > c1rp. c < c1rp: Full in Both,
c ∈ (c1rp,c2rp): Full Pre, Medium Post, c ∈ (c2rp,c1): Full Pre, Exclusive Post,
c ∈ (c1,c2): Medium Pre, Exclusive Post, c > c2: exclusive Pre and Post *)

(* Check Δp for Med to Med *)

FullSimplify[pmed < pmedrp,
 {0 < α < alphatilde, 1 > μ > 0, rhotilde < ρ < rhotilderp, u > 0, Δ > 0}]
```

$\alpha + \mu < 1$

```
(* Check Δp for Excl to Med *)

FullSimplify[pexcl < pmedrp,
 {0 < α < alphatilde, 1 > μ > 0, rhotilde < ρ < rhotilderp, u > 0, Δ > 0}]
```

$\mu < 1 + \alpha\,(-2 + \mu)$

```
(* Check Δq for Excl → Excl *)

FullSimplify[qexcl > qexclrp,
 {alphatilde > α > 0, rhotilderp > ρ > rhotilde, 1 > μ > 0, u > 0, Δ > 0}]
```

$1 + \mu\,\rho > \mu + 2\,\rho$

```
(* *)

(* 1c: ρ > rhotilderp > rhotilde. c1 and c2 determine pricing in baseline,
c3rp in return policy *)

FullSimplify[c3rp < c2, {0 < α < alphatilde, rhotilderp < ρ < 1, 1 > μ > 0, u > 0, Δ > 0}]
```

True

```
FullSimplify[c3rp < c1, {0 < α < alphatilde, rhotilderp < ρ < 1, 1 > μ > 0, u > 0, Δ > 0}]
```

$(-1 + \rho)\,\left(1 + \mu + \mu^2\,(-1 + \rho) + \rho - 2\,\mu\,\rho\right) + \alpha\,(1 + \mu - \mu\,\rho) < 0$

```
(* Two possible scenarios *)

(* 1.c.i:   c2 > c3rp > c1. c < c1: Full in Both,
c ∈ (c1,c3rp): Medium Pre, Full Post, c ∈ (c3rp,c2): Medium Pre,
Exclusive Post, c > c2: Exclusive Pre and Post *)

(* 1.c.ii:  c2 > c1 > c3rp. c < c3rp: Full in Both,
c ∈ (c3rp,c1): Full Pre, Exclusive Post, c ∈ (c1,c2): Medium Pre,
Exclusive Post, c > c2: Exclusive Pre and Post *)

(* Check Δq for Excl → Excl *)

FullSimplify[qexcl > qexclrp,
 {alphatilde > α > 0, 1 > ρ > rhotilderp, 1 > μ > 0, u > 0, Δ > 0}]
```

$1 + \mu\,\rho > \mu + 2\,\rho$

- Case 2: $\alpha$ > alphatilde which implies rhotilde > rhotilderp

```
(* 2a: ρ < rhotilderp. c3 determines pricing in baseline,
c1rp and c2rp in return policy *)
```

```
FullSimplify[c3 > c1rp,
 {1 > α > alphatilde, 0 < ρ < rhotilderp < rhotilde, 1 > μ > 0, u > 0, Δ > 0}]
```

$\alpha\,(-3+\mu\,(4+\mu\,(-1+\rho)-3\,\rho)+\rho) > (-2+\mu)\,(1+\mu\,(-1+\rho))$

```
FullSimplify[c3 < c2rp,
 {1 > α > alphatilde, 0 < ρ < rhotilderp < rhotilde, 1 > μ > 0, u > 0, Δ > 0}]
```

$\Delta\left(-1+\mu+\alpha\left(1-\mu+\frac{1}{(-1+\mu)\,(-1+\rho)}+\frac{1}{-1+\mu-\mu\,\rho}\right)\right) < 0$

```
(* c3 can lie anywhere w.r.t both c1rp and c2rp (where we have c1rp < c2rp) *)

(* Three possible scenarios *)

(* 2.a.i:    c3 > c2rp > c1rp. c < c1rp: Full in Both,
c ∈ (c1rp,c2rp): Full Pre, Medium Post, c ∈ (c2rp,c3): Full Pre,
Exclusive Post, c > c3: Exclusive Pre and Post *)

(* 2.a.ii:   c2rp > c3 > c1rp. c < c1rp: Full in Both,
c ∈ (c1rp,c3): Full Pre, Medium Post, c ∈ (c3,c2rp): Exclusive Pre,
Medium Post, c > c2rp: exclusive Pre and Post *)

(* 2.a.iii:  c2rp > c1rp > c3. c < c3: Full in Both,
c ∈ (c3,c1rp): Exclusive Pre, Full Post, c ∈ (c1rp,c2rp): Exclusive Pre,
Medium Post, c > c2rp: exclusive Pre and Post *)

(* Check Δp for Excl → Med *)

FullSimplify[pexcl > pmedrp, {1 > α > alphatilde, 1 > μ > 0, u > 0, Δ > 0}]
```

True

```
(* Check Δq for Excl → Excl *)

FullSimplify[qexcl > qexclrp,
 {0 < ρ < rhotilderp, 1 > α > alphatilde, 1 > μ > 0, u > 0, Δ > 0}]
```

True

```
(* *)

(* 2b: rhotilde > ρ > rhotilderp. c3 determines pricing in baseline,
c3rp in return policy *)

FullSimplify[Reduce[c3rp > c3],
 {1 > rhotilde > ρ > rhotilderp > 0, 1 > α > alphatilde, 1 > μ > 0, u > 0, Δ > 0}]
```

$(2+\mu\,(-1+\rho)-\rho)\,(\alpha+\mu^2-\alpha\,\mu^2+(-1+\alpha)\,(-1+\mu)\,\mu\,\rho) > 1+\rho$

```
(* Can lie either way *)

(* 2.b.i:    c3 > c3rp. c < c3rp: Full in Both, c ∈ (c3rp,c3): Full Pre,
Exclusive Post, c > c3: Exclusive Pre and Post *)

(* 2.b.ii:   c3 < c3rp. c < c3: Full in Both,
c ∈ (c3,c3rp): Exclusive Pre, Full Post, c > c3rp: Exclusive Pre and Post *)

(* Check Δq for Excl → Excl *)
```

```
FullSimplify[qexcl > qexclrp,
 {rhotilderp < ρ < rhotilde, 1 > α > alphatilde, 1 > μ > 0, u > 0, Δ > 0}]
```

True

```
(* *)

(* 2c: ρ > rhotilde > rhotilderp. c1 and c2 determine pricing in baseline,
c3rp determines pricing in return policy *)

FullSimplify[c3rp < c2,
 {1 > α > alphatilde, 1 > ρ > rhotilde > rhotilderp > 0, 1 > μ > 0, u > 0, Δ > 0}]
```

$(-1+\mu)\ (\alpha\ (-1+\mu)\ (-1+\mu\ (-1+\rho))\ (-1+\rho)+\rho\ (\mu+\rho-\mu\,\rho)) < \rho$

```
FullSimplify[c3rp < c1,
 {1 > α > alphatilde, 1 > ρ > rhotilde > rhotilderp > 0, 1 > μ > 0, u > 0, Δ > 0}]
```

$(-1+\rho)\ (1+\mu+\mu^2\ (-1+\rho)+\rho-2\,\mu\,\rho)+\alpha\ (1+\mu-\mu\,\rho) < 0$

```
(* c3rp can lie above anywhere w.r.t. both c1 and c2 *)

(* Three possible Scenarios *)

(* 2.c.i:   c2 > c3rp > c1. c < c1: Full in Both,
c ∈ (c1,c3rp): Medium Pre, Full Post, c ∈ (c3rp,c2): Medium Pre,
Exclusive Post, c > c2: Exclusive Pre and Post *)

(* 2.c.ii:  c2 > c1 > c3rp. c < c3rp: Full in Both,
c ∈ (c3rp,c1): Full Pre, Exclusive Post, c ∈ (c1,c2): Medium Pre,
Exclusive Post, c > c2: Exclusive Pre and Post *)

(* 2.c.iii: c3rp > c2 > c1. c < c1: Full in Both,
c ∈ (c1,c2): Medium Pre, Full Post, c ∈ (c2,c3rp): Exclusive Pre,
Full Post, c > c3rp: Exclusive Pre and Post *)

(* Check Δq for Excl → Excl *)

FullSimplify[qexcl > qexclrp,
 {1 > α > alphatilde, 1 > ρ > rhotilde > rhotilderp > 0, 1 > μ > 0, u > 0, Δ > 0}]
```

$1+\mu\,\rho > \mu+2\,\rho$

```
(* Overall all 9 cases are possible,
including all possible ambiguities regarding Δp and Δq where relevant *)
```